Exploring Issues in Mathematics Education

An Evaluation of the Numeracy Project for Years 7–10, 2002

Kathryn C. Irwin
School of Education, University of Auckland: Te Whare Wānanga o Tamaki Makaurau

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Author: Kathryn C. Irwin, School of Education, University of Auckland: Te Whare Wānanga o Tamaki Makaurau
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Executive Summary

This report provides an evaluation of the effectiveness of the New Zealand Numeracy Project for years 7–10 in 2002.

The Numeracy Project is a professional development project for teachers that aims to increase students' competence in numeracy by improving the strategies that they use for addition and subtraction, multiplication and division, and proportion problems, and improving their knowledge of the number system, place-value, fractions, decimals, and percentages. Teachers interview students at the start and end of the year to identify the stage they have reached on nine scales designed to assess each of these aspects of numeracy.

The project was developed for teachers of children from years 1–3. It has been extended for use in upper primary school. The order of the stages remains the same for students in years 7–10.

The key findings of this evaluation and the recommendations based on those findings are as follows.

1. The Numeracy Project is associated with improvements in students' performance. Between 30% and 57% of students (according to the scale) gained at least one stage on the knowledge and strategy scales. These percentages are comparable to those for 2001. In addition, in a separate study based on a smaller sample, students in the project did significantly better than students not in the project on a test of pre-algebraic manipulation in arithmetic.

   1a. Recommendation: Facilitators and teachers should continue to work toward pre-algebraic understanding of the arithmetic in the project.

2. The three scales related to reading numbers and knowing their order are easy for this age range. The scale of additive strategies is also easy for many students in this age range.

   2a. Recommendation: Facilitators should encourage teachers to begin by assessing students in this age range only on the more difficult items on these scales, and teach these skills only to those that do not do well on the assessment.

3. Many of these students continue to use additive strategies for multiplication problems.

   3a. Recommendation: Facilitators and teachers should make multiplicative strategies the main focus for these students.

   3b. Recommendation: School administrators should use the multiplicative skill scale as the main indicator of numeracy for these students.

4. Students in low-decile secondary schools were especially unlikely to progress to using multiplication to solve multiplication problems. They started at lower stages than other students did and few moved to using multiplicative strategies. This is a reflection of the project's expectation that older students should move up through the same stages that young children do.

   4a. Recommendation: The Ministry of Education should experiment with modifying the programme for secondary school students in an attempt to enable remedial students to move directly to multiplication.

5. Moving between Stages 2 and 4 is generally easier than moving between Stages 4 and 7.

   5a. Recommendation: School administrators should not expect students to make even progress through all stages.

6. Teachers found that the assessment forms used in 2002 were too long and that having three different forms did not allow them to fully assess their students. They made decisions that may have been appropriate for teaching but that made evaluation less reliable.

   6a. Recommendation: The Ministry of Education should develop a single assessment form for these students. Teachers should start at the point on the form that they think corresponds to the student's ability and then move forward or back as appropriate.

   6b. Recommendation: Facilitators should encourage teachers to use only the scales that they feel are appropriate for each student, rather than assessing all students on all scales.

   6c. Recommendation: The Ministry of Education should require teachers to enter only their final assessments in the national database. This would allow comparison over time, for evaluative purposes.

Chapter 1 – Introduction

This is the second evaluation of the New Zealand Numeracy Project for students in years 7–10 (ages 11–14). See the first report on the effectiveness of this project (Irwin and Niederer, 2002) for these levels in 2001.

The background, rationale, relation to other numeracy projects, and results from previous years can be found in Higgins (2001, 2002), Hughes (2002), Irwin and Niederer (2002), and Thomas and Ward (2001, 2002).

There were several differences between the project in 2002 and the project in 2001. One was that, while in 2001 six schools reported data for years 7 and 8, in 2002 over 700 schools reported data for these years. In 2002, some schools recorded results for only one or two students at these levels, and in other schools there were results for several hundred students. There was also a small increase in the number of secondary schools using the project in 2002, from 10 schools reporting initial and final data in 2001 to 14 schools reporting these data in 2002.

The project itself kept to the same theoretical framework, but scales were separated or added, and the assessment, on which most of this evaluation is based, was put in three separate booklets or forms. Use of separate forms and some of the changes in items were intended to ease administration for teachers and to match teaching intentions. It made evaluation less reliable. For example, one scale went from Stage 0–4 on Form A, from Stage 0–6 on Form B, and from Stage 4–8 on Form C. Some scales appeared on one form and not on others. Thus a student's opportunity to show the extent of their progress was limited by the form or forms of assessment that they were assessed with.

Teachers of students at the levels discussed in this report found the assessment considerably more time-consuming than in 2001. To manage this requirement for extra time, they appear to have made choices that were probably valid from a teacher's viewpoint but that made an evaluator's task more difficult. For example, some teachers and facilitators reported that if they thought a student had made marked progress, they used a more advanced form. Some teachers appeared not to have reassessed on a scale that they knew students to be doing well on. Students not assessed on the same scale at both initial and final times had to be dropped from the analysis of gains.

For secondary schools, 2002 was an unusual year. Many teachers using the Numeracy Project taught at year 11 as well as at years 9 and 10. There was a major change in assessment for year 11 that required the attention of teachers, with the introduction of the National Certificate of Educational Achievement. Attention to this innovation decreased the time available to teachers for becoming familiar with the Numeracy Project. In addition, Collective Employment Contract negotiations were unresolved for several months. Schools took industrial action, with rolling strikes for different classes. One head of a mathematics department reported that constant reminders in the

press and from their union representatives that teachers were undervalued and overworked had a negative effect on her teachers' morale.

Some schools also chose which classes they would assess using the project assessment forms. For example, teachers of year 10 appeared to use the project primarily for their least skilled students. In one case, the head of mathematics of a school reported that the project had been used with all year 10 students, but only the lower bands did the reassessment. Thus results for year 10 should not be seen as representative of student achievement in New Zealand.

The two intermediate schools that were in the project for the second year entered the results from the end of year 7 in 2001 as the assessment for the beginning of 2002. This was logical. However, the changes in the items and scales and differences in teachers' judgement made the assessment of students' gain over 2002 open to question.

In 2001, the scoring sheets did not allow teachers to indicate that students scored at a lower stage on the final assessment than on the initial assessment. In 2002 this restriction was taken off. The overall results showed that between 1% and 3% of students decreased on all scales. This may be realistic, with students being unable to demonstrate a skill on the second occasion that they had demonstrated on the first occasion. However, on some scales and for some schools this percentage of students scoring at a lower level on the second assessment than on the first ranged from 17% to 24%. This may be the result of differences in evaluation of responses. On some occasions, the decrease may indicate an error either in entry or coding. For example, on one scale, several students who scored at Stage 5 initially scored at Stage 2 on the final assessment. This is another indicator of the need for caution in interpreting small changes in increases or decreases in score.

Scales and Stages

Scales

The assessment included nine scales, although not all scales were represented on all assessment forms. There were three strategy scales and six knowledge scales. Questions for each scale are given in Appendix A. This is taken from The Diagnostic Interview (retrieved 3 December 2003 from http://www.nzmaths.co.nz/Numeracy/Numeracy_PDFs/diagint.pdf).

The strategy scales, dealing with computation, were:

Strategies for addition and subtraction. These operations are called "additive strategies" in the figures in this report, as well as in general writing about this field. In assessing this scale, students were given addition and subtraction problems to do. The teacher noted whether the student: counted all objects to obtain an answer; counted on or counted back from one of the numbers; had a "part-whole" strategy in which they broke up one of the numbers being added or subtracted parts to make the problem easier; or had a range of such part-whole strategies. On Form A, this scale went from Stage 0–4, on Form B it went from Stage 4–5, and on Form C it included only Stage 6.

Strategies for multiplication and division. Together, these are referred to as "multiplicative strategies". In assessing this scale, the teacher noted whether students completed a problem that could have been solved using multiplication by using a counting strategy or by repeated addition (for example, if the student knows that 13 x 7 can be solved by multiplying 10 x 7 and then adding 7 three more times: "77, 84, 91"), whether students derived the answers to unknown multiplication questions from known facts in addition and multiplication (for example, 32 x 7 is the same as 30 x 7 plus 2 x 7), or whether they used a range of part-whole strategies. This scale was on Form B with Stages 2–6 and on Form C with stages 4–7.

Strategies for solving ratio and proportional problems. This scale took students into fractional, ratio, and proportional problems. In this report, it is referred to as "proportional strategies". These problems required similar skills to those needed to solve multiplicative problems, but at the upper stages they also required at least two multiplicative processes, such as the division and multiplication required to find three quarters of 24. At the lower stages, the student is asked to find a fraction of a whole number, like one quarter of 24. At the more advanced stages, students are required to find the relationship between two numbers and then apply this relationship to a third number (for example, if 16 bags of apples weigh 10 kg, what would be the weight of 24 bags of apples?). This scale appeared on assessment Form B for Stages 1–6 and on Form C for Stage 5–8.

The knowledge scales were:

Whole number identification. In assessing this scale, students were asked to identify printed numerals. The numbers ranged from two-digit to six-digit figures. This scale was only on Form A.

Forward number word sequence. This scale is often referred to as "FNWS". In assessing this scale, students were asked to name the number directly following a written numeral. This appeared on assessment Forms B and C.

Backward number word sequence. This scale is often referred to as "BNWS". In assessing this scale, students were asked to name the number directly before a written numeral. This appeared on assessment Forms B and C.

Knowledge of fractions. In assessing this scale, students were required to match fractions to samples from a pie diagram, to read unit fractions less than one (1/2, 1/4, 1/3), to indicate the meaning of a fraction greater than one, and to order fractions with different numerators and denominators. This scale appeared on Form B for Stages 2–6 and on Form C for Stages 2–8.

Knowledge of decimals and percentages. Assessing this scale required students to read, order, and round decimals and translate between decimals and percentages. In 2001, some of these skills were included in the scale for knowledge of fractions. It was separated from fraction knowledge in 2002. It appeared only on Form C, Stages 4–8.

Knowledge of grouping and place-value. In early stages of this scale, students were required to tell how many dots were in groups of five and ten. In Stages 4–6, they were required to name the number of 10s in numbers between two and five digits long

and to give the number of 100s in numbers from six to seven digits long. At Stages 7 and 8, students were required to name the tenths and hundredths in numbers that included both whole numbers and decimal fractions. Form A went from Stage 0–4, Form B went from Stage 0–6, and Form C went from Stage 4–8.

Stages

Stages are defined in relation to strategy scales. The low starting levels reflect the fact that the project was initially intended for young children just starting schooling. The stages were the same as in 2001.

Stage 0. Pre-counting. Students at this level cannot count a small group of objects.

Stage 1. Count from one on materials. Students at this stage can count and form a set of up to 10 objects by counting each one. They cannot solve simple adding problems by joining these sets.

Stage 2. Adding by counting from one with materials. These students can add four counters and two counters by counting all of them.

Stage 3. Counting from one by imagining the objects to be counted. These students use counting but do not need to see objects in order to add.

Stage 4. Advanced counting. Students at this stage solve addition problems by counting on. For example for 8 + 3 they say "8, 9, 10, 11" to get the answer 11.

Stage 5. Early additive part-whole thinking. At this stage students recognise that addition problems can be solved efficiently by breaking up numbers into their component parts. For example, students who do not automatically know that 8 and 5 is 13 can see that 5 can be broken up into 2 and 3, and that since 8 + 2 = 10, 3 more makes 13.

Stage 6. Advanced additive/early multiplicative thinking. Students at this stage use a variety of ways to break up numbers for doing addition problems and may do multiplication problems by using these part-whole addition strategies. For example, they may mentally work out that 63 – 29 can be worked out by thinking that 63 – 30 = 33, and adding one would be 34.

Stage 7. Advanced multiplicative/early proportional thinking. At this stage, students can use their understanding of multiplication to break up numbers. For example, they may realise that 50 x 124 is the same as 100 x 62, so the answer will be 6,200.

Stage 8. Advanced proportional thinking. Students at this stage can use a range of multiplication and division strategies to solve proportion problems. This includes finding a percentage of a whole number. Students who can do this might find 15% of 240 by first finding 10% (24) and then adding half of this (12). When these two percentages are added together, they would get 36 as 15% of 244.

Research Questions

Research questions addressed were:

1. To see if stages on different scales were equivalent and if any differences found suggested different difficulty of scales.

   This is addressed in Chapter 2.

2. To examine the performance and gains of all students and to see if there was a noticeable difference in the results between those schools that were in the project for the first year and those that were in the project for the second year.

   This is addressed in Chapters 3 and 5.

3. To explore the students' ability to identify a general rule for the numerical strategies developed.

   This is addressed in Chapter 4.

4. To explore differences between schools in different deciles in the development of multiplicative strategies in secondary students from schools in the project for the second year.

   This is addressed in Chapter 6.

5. To examine the pattern of attainment over the transition between intermediate and secondary school.

   This is addressed in Chapter 7.

6. To compare the results from 2001 with those from 2002.

   This is addressed in Chapter 8.

7. To explore the views of teachers and facilitators involved in the project.

   This is addressed in Chapter 9.

Not all of the proposed research questions could be addressed because of limitations in the way in which data were recorded and because of the unusual sample of secondary school students. Questions not addressed related to: a) comparison of students who had been in the project in the previous school or year, due to inadequate data provided; b) continued exploration of successful implementation in secondary schools, due to unusual events in the secondary schools; and c) generalisation of skills to other spheres of mathematics, due to the decision to concentrate on a larger study of students' ability to apply a general rule in arithmetic as a precursor to algebra.

Outline of This Report

Chapter 2 uses Rasch analysis to determine the order of difficulty of scales and stages and the relative distance between stages on the different scales. It also demonstrates that tasks were of comparable difficulty on initial and final assessment. Chapters 3 and 5 provide information on the initial and final assessment of students in years 7 and 8 and years 9 and 10 and on the gain made on each of the scales. Chapter 4 provides an analysis of a test of algebraic thinking in arithmetic, given to two schools using the Numeracy Project and to two comparable schools not in the project. This is considered a test of near generalisation. Chapter 6 analyses gains made in multiplicative thinking in two decile 1 schools and two decile 8 or 9 schools that were in the project for the second year. Chapter 7 compares the initial and final assessment of years 7, 8, and 9 on the strategy scales. Chapter 8 provides a comparison of the gains made in strategy skills in 2001 and 2002. Chapter 9 provides a summary of the views of the teachers and facilitators involved in the project.

Chapter 2 – Analysis of the Stages and the Different Scales

Measurement of the Scales and Stages

A Rasch analysis (see, for example, Bond and Fox, 2001) provided estimates both of the comparative difficulty of each scale and of the size of the intervals between each stage of a scale. These two aspects of the scales are described separately. The analysis was undertaken on performance on the final assessment of 13,600 students. The data provided a reasonable fit to the Rasch model: the average mean-squared residuals (scaled to have an expected value of 1) between model and data was 1.05, with a range from 0.86–1.15 for the nine scales. The reasonable fit means that the nine scales belong to a single dimension.

Scale difficulties

Figure 2.1 depicts the comparative difficulty, estimated from the results of the final assessment, of each of the nine scales. The most difficult scale, shown at the top of the figure, was Knowledge of Decimals and percentages; the least difficult scale, shown at the bottom of the figure, was Whole number identification. The scales of Multiplicative strategies and Knowledge of grouping and place-value are shown at the same position in the figure, near the middle of the continuum, because they were equally difficult.

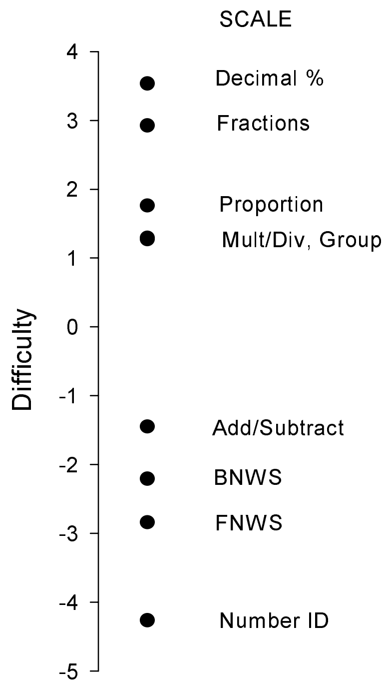


Figure 2.1. Rasch estimates of the relative difficulty of the nine scales.

As can be seen, the scales are not evenly spaced along the continuum of difficulty. The mean of the continuum in Figure 2.1 has been arbitrarily set to zero, and the units of the continuum are log-odds ratios, or logits. In this context, the difference between two scales, in logits, is the natural logarithm of the odds of a student succeeding on one scale rather than the other. Stated differently, a student whose ability equals the difficulty of an item or scale (Rasch analysis specifies the ability of students and the difficulty of the scales on the same continuum) has a 0.5 probability of succeeding on that scale.

According to Rasch theory, the estimated scale difficulties should be independent of the sample from which they were derived. Comparing the scale difficulties estimated on the initial assessment with those estimated on the final assessment provides a partial test of this requirement. Figure 2.2 illustrates the comparison. In this figure, the forty-five-degree diagonal shows identical difficulties of both estimates. The obtained estimates are very similar to each other and do not depart far from the diagonal; however, their standard errors are small, and so small differences can be significant. Although those students taking the final assessment are the same students as those taking the initial assessment, nevertheless they are older at the final assessment and have been exposed to the numeracy program. To that extent, the two samples are different, and thus the requirement can be evaluated.

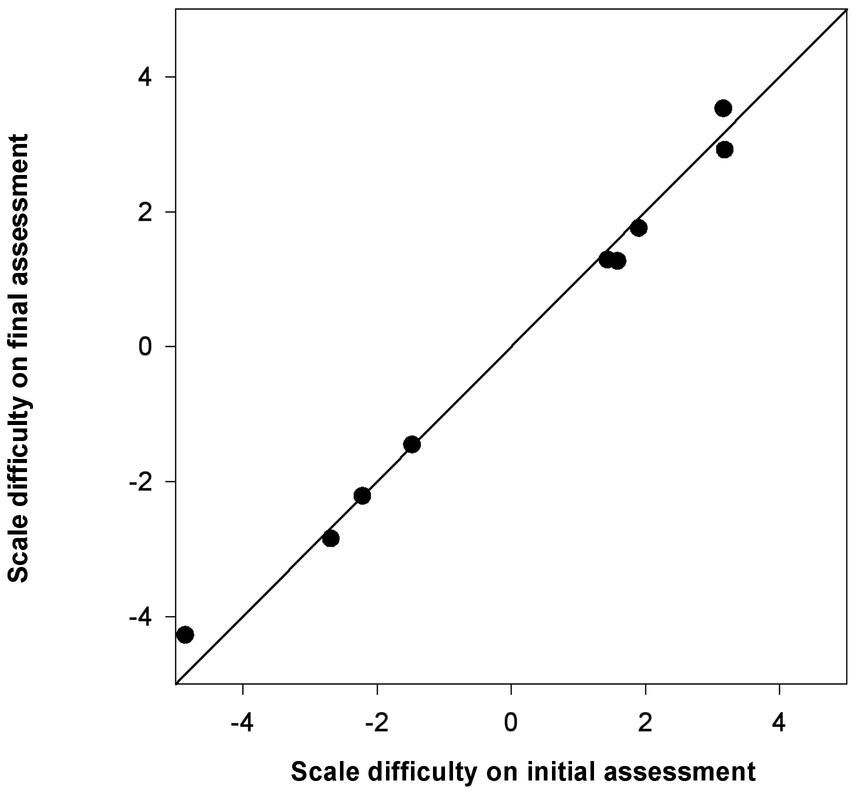


Figure 2.2. Comparison of estimated scale difficulty from the initial and final assessment. The diagonal line represents identical difficulties estimated from each assessment.

Intervals between stages

Figure 2.3 depicts the intervals in logits between adjacent stages, averaged over the nine scales. The figure shows that the average distance between the adjacent stages from 2 to 4 is smaller than the distances between the other stages. The largest interval lies between Stages 5 and 6. The difference between adjacent stages on this continuum is specified as the natural logarithm of the odds of a student of a given ability achieving one stage rather than the other, averaged over all scales.

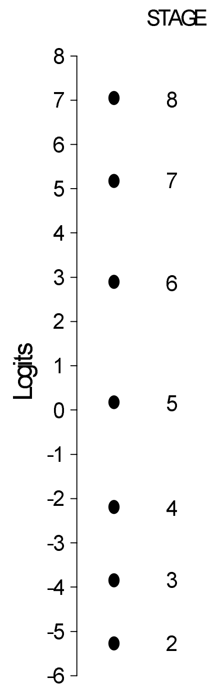


Figure 2.3. Rasch estimates, averaged for all scales, of the interval between stages.

Figure 2.4 shows the interval between stages for each scale and therefore provides a more detailed picture than the average intervals, which are shown in Figure 2.3. In Figure 2.4, the nine scales are arranged in order of difficulty, and the interval between stages is shown for each.

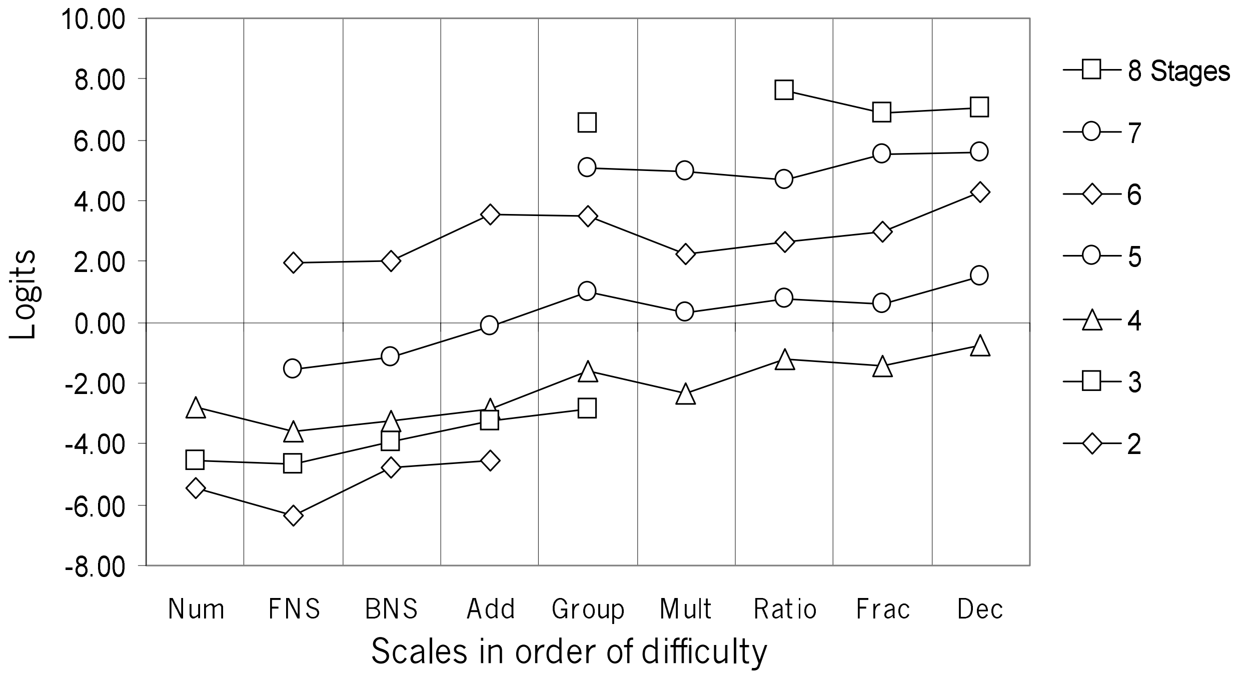


Figure 2.4. Rasch estimates of the interval between stages for each scale.

In Figure 2.4, each stage value is represented by a different symbol. The lines connect the same stage for different scales. The interval between stages is not constant for any scale (nor for their average – see Figure 2.3). Furthermore, the distance between stages depends to some extent on the scale. For example, Stages 2, 3, and 4 are closely spaced on Scale BNS (Backward number word sequence) and more widely separated on FNS (Forward number word sequence). The figure also reveals a tendency for stages to increase in value with increasing scale difficulty. This is most noticeable for Stages 3, 4, and 5. For example, Stage 5 increases in value from -1.55 units for Scale FNS to 1.49 units for Scale Dec (Knowledge of decimals and percentages), a difference of 3.06 logits. In other words, not only are the differences between stages unequal, but so also are the values of a given stage on different scales.

Chapter 3 – Performance of Students in Years 7 and 8

Year 7 and 8 students in this study came from over 700 schools in all parts of New Zealand. It was not possible to tell from the data provided which students came from intermediate schools and which came from full primary schools or area schools.

Decile Levels of the Students' Schools

Schools at all decile levels were represented, with the largest number of students coming from decile 4 schools. The majority of students (54%) came from schools at the lowest four decile rankings. As one intention of the project was to aid lower decile schools first, this distribution was expected.

Table 3.1 Numbers and percentages of year 7 and 8 students from schools in each decile ranking, where given.

Decile	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
1	687	11%	497	9%
2	675	11%	648	12%
3	641	10%	633	11%
4	1278	21%	1307	23%
5	759	12%	705	13%
6	546	9%	375	7%
7	267	4%	217	4%
8	418	7%	425	8%
9	218	4%	273	5%
10	572	9%	390	7%
Not given	157	3%	154	3%
Total	6,218	100%	5624	100%

Only two of the schools that had been involved in the project in 2001 returned data for 2002. These were a decile 4 school and a decile 10 school. Comparison of the performance of these schools with others at the same decile level is provided in the chapter on comparison of schools in the first and second year in the project.

Gender and Ethnicity of Year 7 and 8 Students

There were 5,757 girls (49%) and 6,092 boys (51%) at this age level in the project. See Table 3.2.

Table 3.2 Gender of year 7 and 8 Students in the project in 2002

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Girls	2,973	48%	2781	49%
Boys	3,245	52%	2843	51%

Although the majority of students were of European background, a higher proportion of Māori and Pasifika students were in the sample than would be expected from national statistics. As a high percentage of Māori and Pasifika students attend schools with lower decile ranking, this is not unexpected.

Table 3.3. Numbers and percentages of the given ethnicity of year 7 and 8 students.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
European	3,463	56%	3,245	58%
Māori	1,646	26%	1,452	26%
Pasifika	560	9%	460	8%
Asian	309	5%	293	5%
Other	240	4%	174	3%
Total	6,218	100%	5624	100%

Initial and Final scores on Strategy Scales, Years 7 and 8

The percentages for year 7 and 8 students at each stage are given in Appendix B. The category "NA" indicates that the assessment was not given.

Additive strategies

Nearly all year 7 and 8 students scored at least at the advanced counting stage (Stage 4) while the majority, both initially and finally, scored at the early or advanced part-whole stages (Stage 5 and 6).

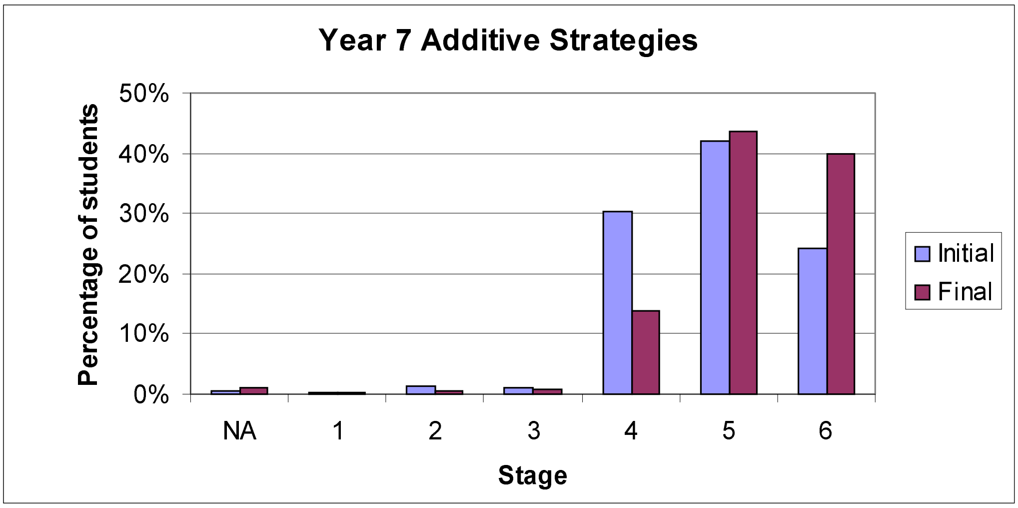


Figure 3.1. Percentages of year 7 students at each stage for additive strategies.

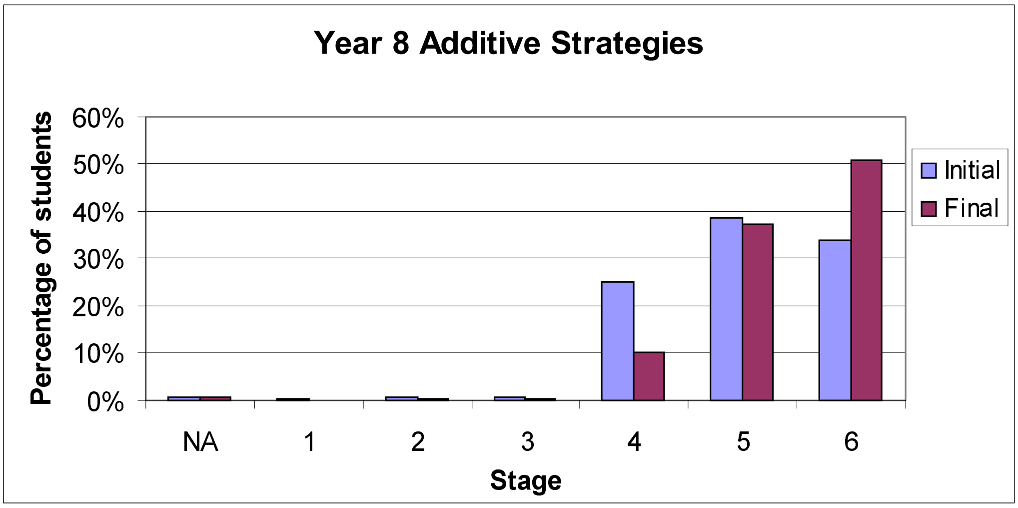


Figure 3. 2. Percentages of year 8 students at each stage for additive strategies.

The pattern of achievement for year 8 students was similar to that for year 7 students, with a somewhat lower percentage of year 8 students using advanced counting (Stage 4) when assessed initially and a higher percentage using advanced part-whole strategies on the final assessment.

Gains for additive strategies

Table 3.4 gives the number of students assessed on initial and final occasions on additive strategies, and the numbers and percentages who remained at the same stage, gained, and did not gain.

Table 3.4. Numbers and percentages of students making gains in additive strategies.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	101	2%	86	2%
Ceiling both times*	1,444	23%	1,810	33%
Gain 0 if could gain*	920	15%	1,735	31%
Gained 1 or more stage*	2,675	44%	1,851	33%
Lost 1 or more stage*	1,067	17%	142	3%

*from those students assessed on both occasions (N for year 7 = 6,107, N for year 8 = 5,538)

There are some doubts about the data for year 7 on gains, as 66 students were listed as moving from Stage 2 to Stage 6 or 7, which is unlikely. Several students initially scored as part-whole thinkers were recorded as using counting strategies in the final analysis.

Move to part-whole strategies for addition

The move to the use of part-whole strategies is the important step on this scale. Table 3.5 shows the percentages of year 7 and year 8 students who used part-whole strategies on both assessments and the percentages of those who came to use these strategies. The fact that year 8 started with fewer part-whole thinkers than there were at the end of year 7 suggests that it was the project, not increasing age or an existing programme, which was responsible for this increase in part-whole thinking.

Table 3.5. Percentages of year 7 and 8 students who used part-whole strategies either both initially and finally or who came to use part-whole strategies during the project.

	Year 7	Year 8
Used part-whole strategies on both assessments	66%	71%
Came to use part-whole strategies during the year	19%	19%

By the end of the year, 85% of the year 7 students and 90% of the year 8 students were recorded as using part-whole methods to solve addition problems. Of the students who were initially designated at using counting strategies for adding, 1,165 (57%) of the year 7s and 924 (64%) of the year 8s moved to using part-whole methods for mental addition.

Multiplicative strategies

A wider range of multiplicative strategies than of additive strategies was used. All students below Stage 6 (early multiplicative) used counting or adding strategies for multiplication questions. While a sizable proportion of both year 7 and year 8 students used advanced counting strategies (Stage 4) for these problems initially, a somewhat larger percentage used additive part-whole strategies. Initially, 34% of the students used multiplicative strategies, and, on the final assessment, 55% of the students used multiplicative strategies.

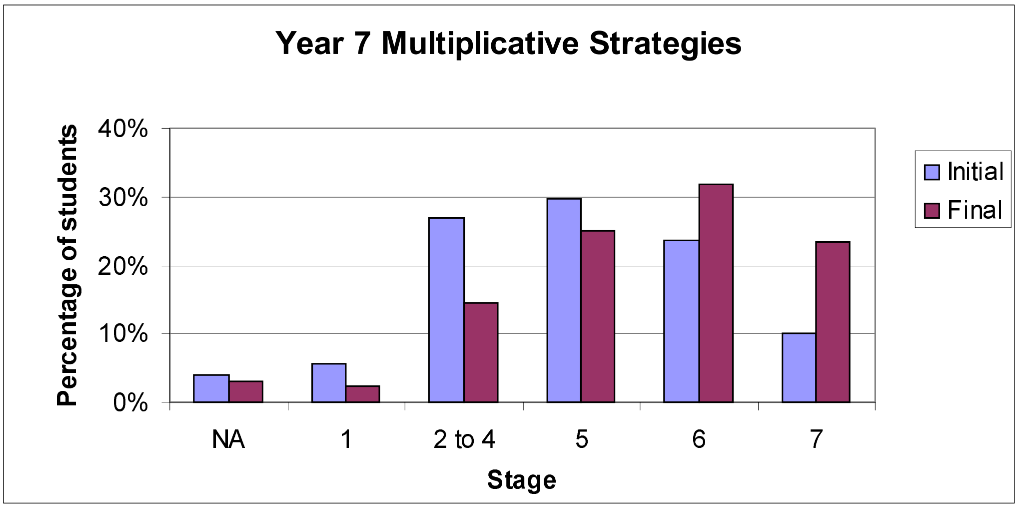


Figure 3.3. Percentages of year 7 students at each stage for multiplicative strategies.

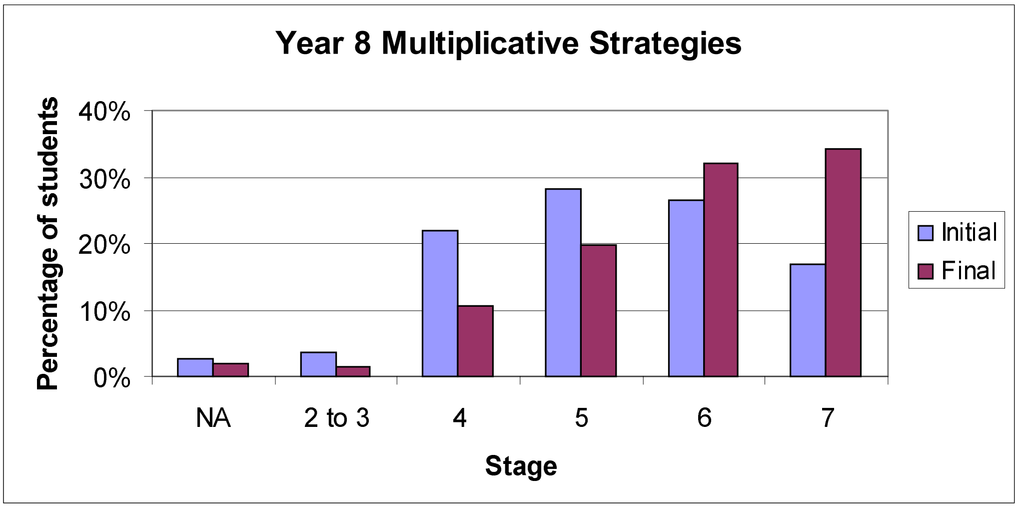


Figure 3.4. Percentages of year 8 students at each stage for multiplicative strategies.

Gains in multiplicative strategies

Table 3.6 gives the number of students assessed on both occasions and the number and percentage who gained and who did not gain in multiplicative strategies.

Table 3.6. Numbers and percentages of students making gains in multiplicative strategies.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	313	5%	207	4%
Ceiling both times*	594	10%	892	16%
Gain 0 if could gain*	2,252	38%	1,707	32%
Gained 1 or more stage*	2,934	50%	2,671	49%
Lost 1 or more stage*	132	2%	147	3%

* from those students assessed on both occasions (N for year 7 = 5,905, N for year 8 =5,417)

About half the students assessed on both occasions gained at least one stage. What is of more interest is the number and percentage who moved to using some multiplicative processes for these problems, as increases in stages which are still additive do not enable students to use the power afforded by dealing with groups of numbers as single entities. Those considered to have reached the multiplicative strategy levels, of those assessed on both occasions, were those who scored initially at Stage 4 or below and scored at Stage 6 or 7 on the final assessment. A higher percentage of year 8 students were already multiplicative thinkers, although not as high as the percentage of year 7 students at the end of the year.

Table 3.7. Percentages of year 7 and 8 students moving from the use of counting or adding strategies to multiplicative strategies during the project.

	Year 7	Year 8
Multiplicative thinkers on both assessments*	33%	44%
Became multiplicative during the year	22%	25%

*from those students assessed on both occasions (N for year 7 = 5,912, N for year 8 =5,417)

Of the year 7 students who used counting or adding methods for multiplication problems initially, 36% came to use multiplicative methods for these problems. Of the year 8 students using adding strategies initially, 41% came to using multiplicative thinking on these problems during the year.

Proportional strategies

Although problems involving ratios and proportions require the same skills that multiplication problems do, the fact that more than one process is usually required appears to make them more difficult for students. On this scale the largest proportion of students used a procedure known as "equal sharing" initially, a procedure more akin to counting than to either additive or multiplicative strategies. Stage 5 requires additive part-whole strategies and Stage 6 requires multiplicative strategies. Only Stages 7 and 8 require early proportional or advanced proportional thinking.

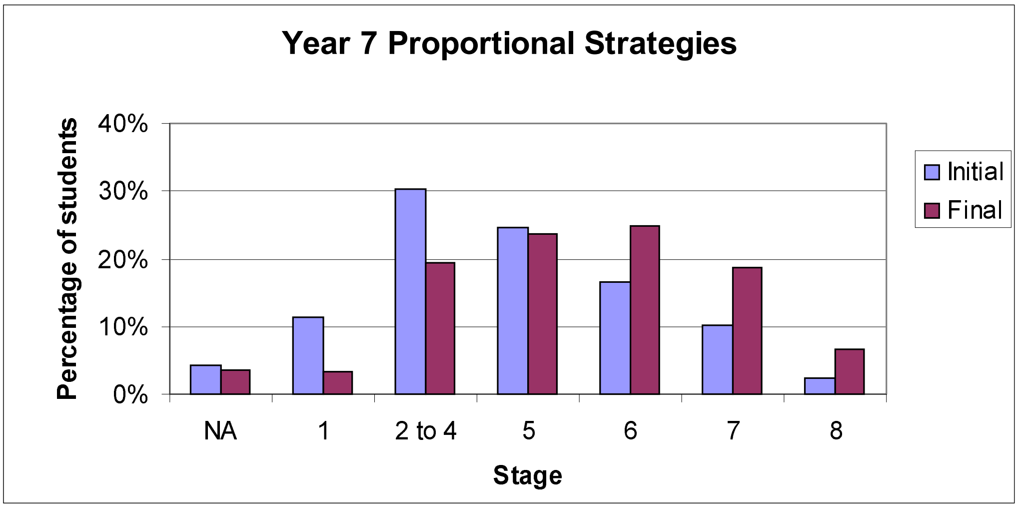


Figure 3.5. Percentages of year 7 students at each stage for proportional strategies.

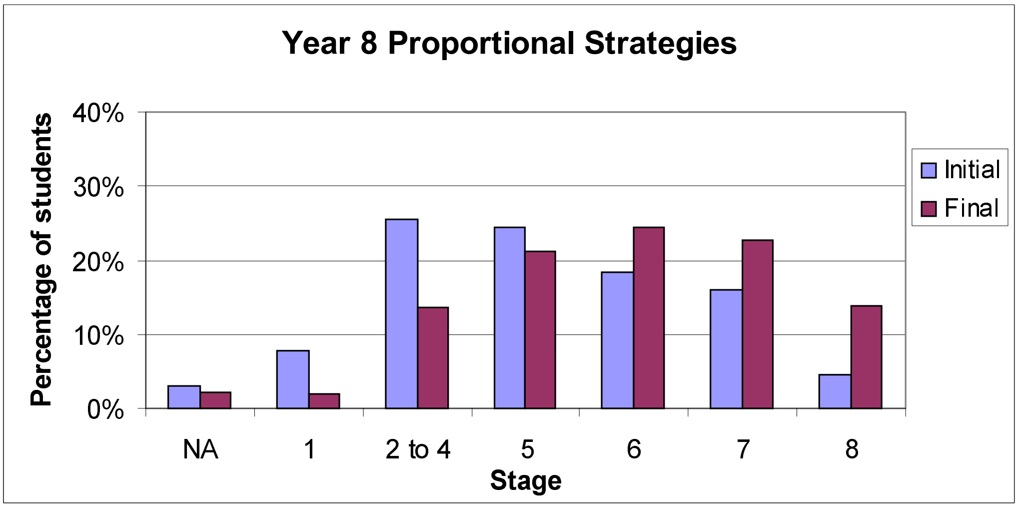


Figure 3.6. Percentages of year 8 students at each stage for proportional strategies.

This was the most difficult of the strategy scales, even if students were given the opportunity to demonstrate strategies at the top levels. It is difficult because

proportional reasoning involves at least two multiplicative processes. The following table shows that, given the opportunity, more than 50% of each year did demonstrate gain.

Only those students who scored at Stage 6 or above used multiplicative strategies, and only those at Stage 7s and 8 (early and advanced proportional) demonstrated the use of more than one multiplicative process to solve these problems mentally. However, it is important to note that only students assessed on Form C had the opportunity to be scored at the stages that indicate proportional thinking. Where students did move from non-proportional thinking to proportional thinking, this could have been because they were assessed on Form B for the initial assessment and Form C for the final assessment.

Table 3.8. Numbers and percentages of students making gains in use of proportional strategies.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	355	6%	207	4%
Ceiling both times*	151	3%	151	2%
Gain 0 if could gain*	2,493	42%	2,478	41%
Gained 1 or more stage*	3,125	53%	3,253	54%
Lost 1 or more stage*	101	2%	103	2%

*from those students assessed on both occasions (N for year 7 = 5,870, N for year 8 = 5,417)

The important step on this scale is becoming a proportional thinker.

Table 3.9. Percentages of year 7 and year 8 students who were proportional thinkers or became so during the project.

	Year 7	Year 8
Proportional thinkers on both assessments*	15%	21%
Became proportional thinkers in the year*	14%	17%

*from those students assessed on both occasions (N for year 7 = 5,870, N for year 8 = 5,417)

Of those students who were not recorded as proportional thinkers on both occasions, 18% of the year 7 students came to use proportional thinking and 21% of the year 8 students came to use proportional thinking by the time of the second assessment.

Interpretation of this information is hampered by the limited number of students who were given the opportunity to show early and advanced proportional thinking, which was assessed only on Form C.

Initial and Final Scores on Knowledge Scales, Years 7 and 8

Number identification

This test was given only to students assessed on Form A. Only 3% of each year group was tested twice on this scale. Of that 3%, 67% of year 7 and 71% of year 8 students were at the top level on both occasions, and 18% of year 7 students and 8% of year 8 students gained at least one stage. See Appendix B for detailed results.

Forward number word sequence

This test was on Forms A, B, and C. Form A went only to Stage 4. Most students in both years could give the number after a given three-digit number (Stage 5) and after a given six-digit number (Stage 6).

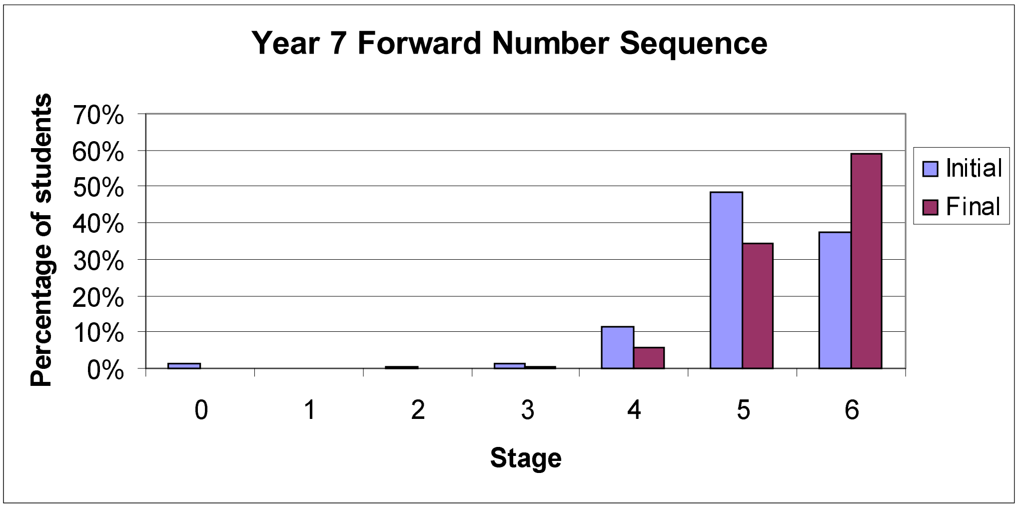


Figure 3.7. Percentages of year 7 students at each stage for FNWS.

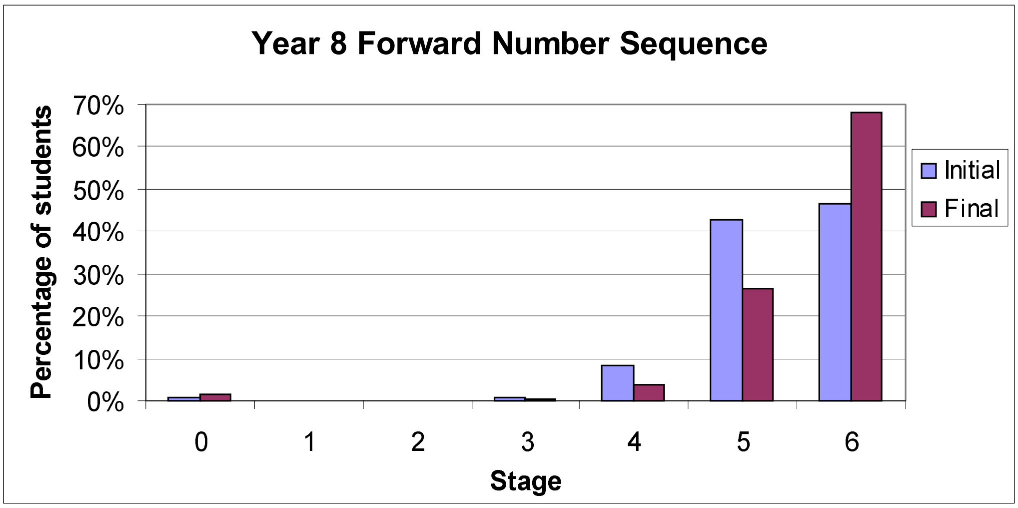


Figure 3.8. Percentages of year 8 students at each stage for FNWS.

Most students in both years were at Stages 5 and 6 – able to give the number following any three-digit or six-digit numeral.

Table 3.10. Numbers and percentages of year 7 and 8 students making gains in FNWS

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	146	2%	119	2%
Ceiling both times*	2,191	36%	2,472	45%
Gain 0 if could gain*	1,898	31%	1,273	23%
Gained 1 or more stage*	1,822	30%	1,595	29%
Lost 1 or more stage*	168	3%	163`	3%

* from those students assessed on both occasions (N for year 7 = 6,072, N for year 8 = 5,055)

A smaller percentage of those who could gain on this scale did so than the percentage that gained on the strategy scales. This could be because teaching this skill was not emphasised.

Backward number word sequence

This assessment was done at the same time as FNWS, on Forms B and C. Most students scored at the top two stages, as they did on FNWS.

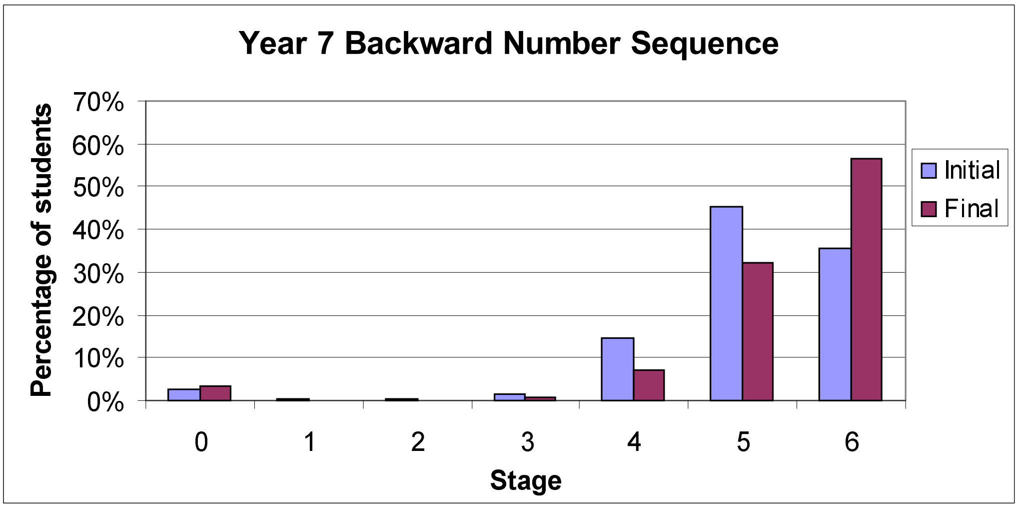


Figure 3.9. Percentages of year 7 students at each stage for BNWS.

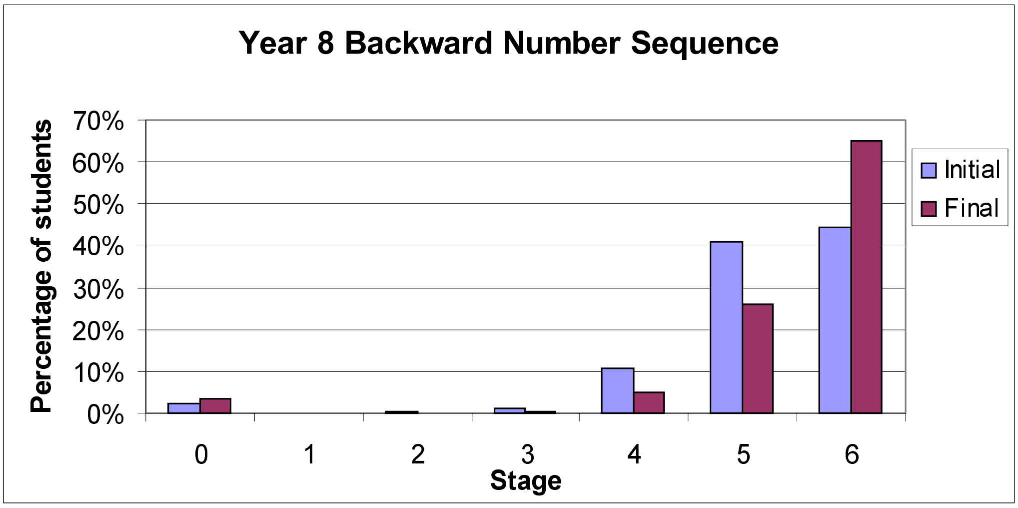


Figure 3.10. Percentages of year 8 students at each stage for BNWS.

Results for this scale were similar to those for the forward word number sequence scale. Most students were at the top two levels, being able to give the numeral before a given three-digit or six-digit numeral.

Table 3.11. Numbers and percentages of year 7 and 8 students making gains in BNWS.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	286	5%	267	5%
Ceiling both times*	2072	35%	2320	43%
Gain 0 if could gain*	1789	30%	1260	24%
Gained 1 or more stage*	1912	32%	1606	30%
Lost 1 or more stage*	190	3%	172	3%

* from those students assessed on both occasions (N for year 7 = 5,932, N for year 8 = 5,357)

The somewhat higher number of students not given this test on both occasions may have been due to teachers choosing not to ask these questions of students whom they saw as competent. The percentages of year 7 and year 8 students who made gains were similar to those for FNWS.

Knowledge of fractions

Form B assessed Stages 2–6 on this scale and Form C assessed Stages 2–8. Thus the top two stages could only be assigned to students assessed on Form C. There is no way of knowing if the students who scored at the top two levels in the final assessment were given the opportunity to score at these levels initially.

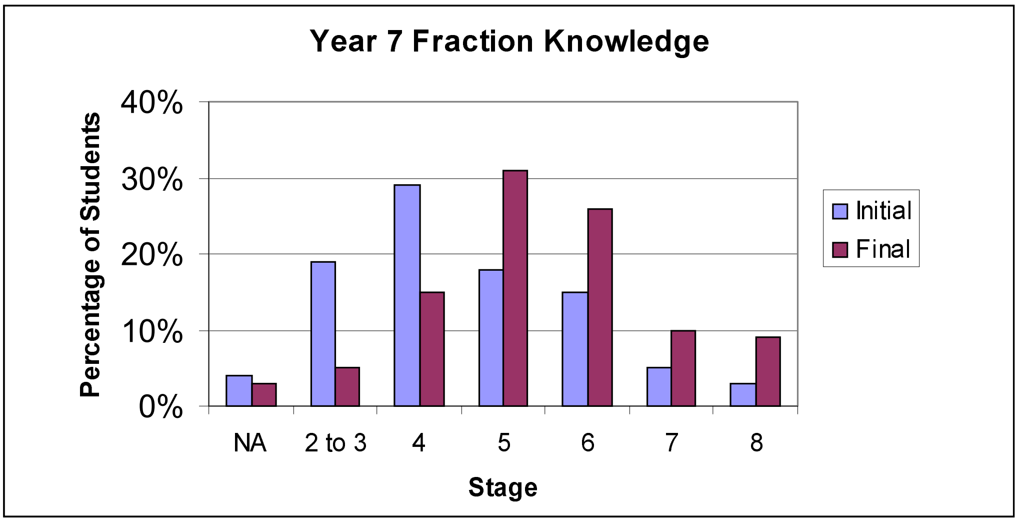


Figure 3.11. Percentages of year 7 students at each stage for knowledge of fraction words.

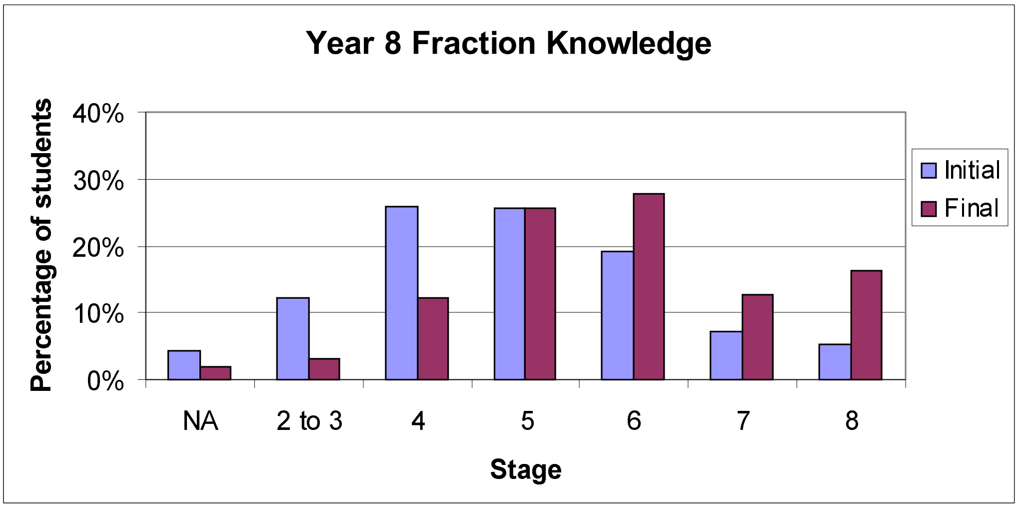


Figure 3.12. Percentages of year 8 students at each stage for knowledge of fraction words.

A major finding on this scale was the percentage of students scoring at Stages 2–4 initially. It was not possible to score at a lower stage on this scale. These students either did not know what fraction symbol stood for a region of a circle (Stage 2–3) or could not order fractions from smallest to largest (Stage 4). Results on the second assessment suggest that these skills had been taught and learned.

Table 3.12. Numbers and percentages of year 7 and 8 students making gains in knowledge of fractions.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	464	7%	298	5%
Ceiling both times*	325	6%	279	5%
Gain 0 if could gain*	1,606	28%	1,711	32%
Gained 1 or more stage*	2,661	46%	3,211	60%
Lost 1 or more stage*	1,169	20%	175	3%

* from those students assessed on both occasions (N for year 7 = 5,754, N for year 8 = 5,326)

A higher percentage of year 8 than of year 7 students gained at least one stage on this scale. The percentage of year 8 students gaining on this scale was higher than the percentage of students gaining on any of the strategy scales.

Knowledge of decimals and percentages

This scale was on Form C only. Some teachers commented, in interviews and on questionnaires, that some assessment of these skills was needed on Form B.

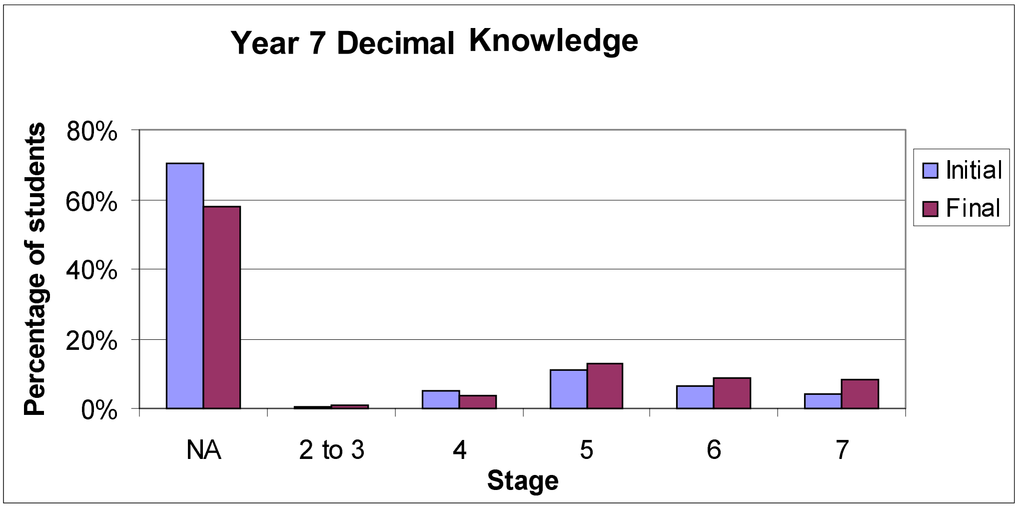


Figure 3.13. Percentages of year 7 students at each stage for knowledge of decimals and percentages.

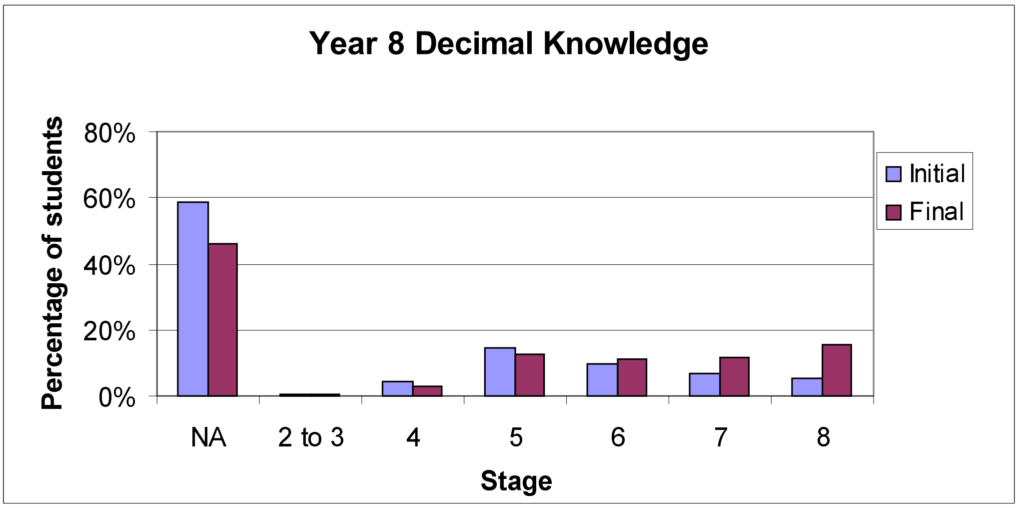


Figure 3.14. Percentages of year 8 students at each stage for knowledge of decimals and percentages.

Although some gain was shown on this scale, the small proportion of students asked about this area makes it difficult to say much about this increase. Some teachers reported giving Form B initially and Form C on the final occasion.

Table 3.13. Numbers and percentages of year 7 and 8 students making gains in knowledge of decimals and percentages.

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	4,441	71%	3,409	61%
Ceiling both times*	135	8%	271	12%
Gain 0 if could gain*	676	38%	711	32%
Gained 1 or more stage*	925	52%	1,171	53%
Lost 1 or more stage*	47	3%	62	3%

* from those students assessed on both occasions (N for year 7 = 1,777, N for year 8 = 2,215)

About half of the students given this scale on both occasions gained one or more stages.

Knowledge of grouping and place-value

This was assessed on all three forms, but Form A covered Stages 0–4, Form B covered Stages 0–6, and Form C covered stages 4–8. It was not possible to tell on what forms students had been assessed, except that all students judged to be at Stages 7 or 8 were assessed on Form C, at least on the final assessment.

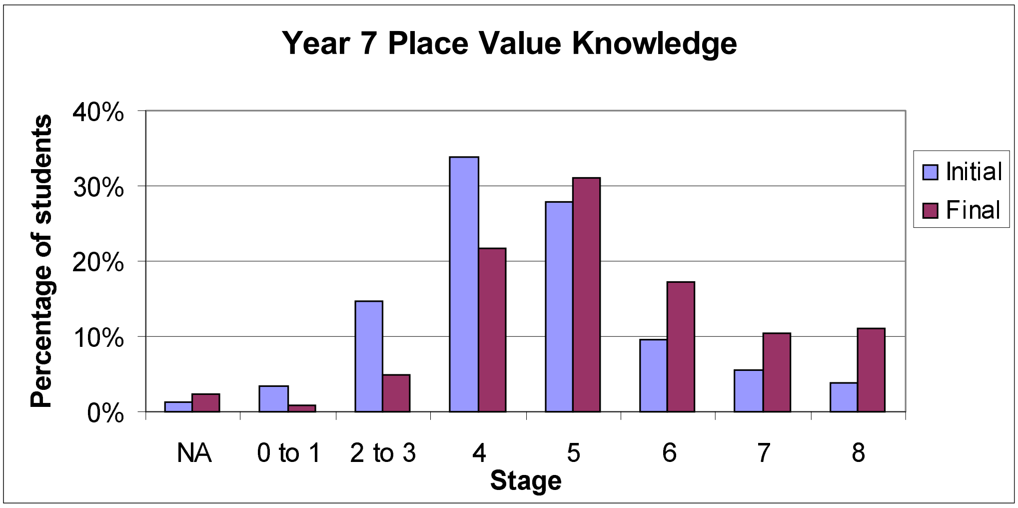


Figure 3.15. Percentages of year 7 students at each stage for knowledge of grouping and place-value.

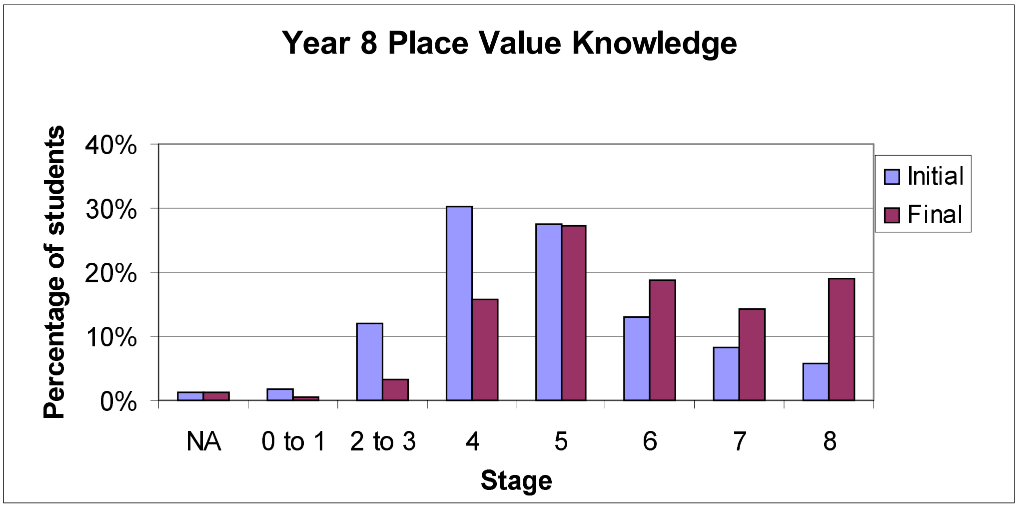


Figure 3.16. Percentages of year 8 students at each stage for knowledge of grouping and place-value.

Students scoring at Stage 4 on this scale could recognise groups of 10 and tell how many were in six sets of 10. Students scoring at Stage 5 were able to give the numbers of 10s in any number of up to five digits. Those scoring at Stage 6 could give the number of 100s in a six- or seven-digit number. Those scoring at Stages 7 and 8 could

give the number of tenths and hundredths in numbers containing a whole number and a decimal fraction.

Table 3.14. Numbers and percentages of year 7 and 8 students making gains in knowledge of grouping and place-value

	Year 7 Number	Year 7 Percentage	Year 8 Number	Year 8 Percentage
Not assessed twice	169	3%	124	2%
Ceiling both times*	214	4%	309	6%
Gain 0 if could gain*	2,345	39%	1,777	32%
Gained 1 or more stage*	3,331	55%	3,253	59%
Lost 1 or more stage*	165	3%	175	3%

*from those students assessed on both occasions (N for year 7 = 6,049, N for year 8 = 5,500)

Over half of the students from each class who were assessed on both occasions gained a stage.

Comparison of Gains for Schools That Entered the Project in 2001 and 2002

It was not always possible to tell, with any certainty, which students had been in the project in the previous year. However, there were two large intermediate schools that were known to be in the project in both 2001 and 2002. The table below compares the gains of their year 8 students in 2002 with all other students of the same decile ranking.

The year 8 students from one decile 10 school were compared with all year 8 students from other decile 10 schools. There were 11 of these other schools. The number of year 8 students per school varied from four to 70.

While these data give a picture of likely differences between schools in the first and second year in the project, the facilitators from both of the schools in the second year of the project reported that irregularities in assessment or reporting meant that these data would need to be interpreted with caution.

Table 3.15. Comparison of gains made by year 8 students from decile 10 schools on strategy scales in the project for the first and second years.

	Additive 1st year in 2002	Additive 2nd year in 2002	Multiplicative 1st year in 2002	Multiplicative 2nd year in 2002	Proportional 1st year in 2002	Proportional 2nd year in 2002
Number assessed twice	223	145	226	143	225	143
Ceiling both times*	47%	67%	26%	68%	7%	26%
Gained 0*	22%	14%	34%	4%	40%	30%
Gained 1 or more stage*	31%	19%	41%	28%	52%	42%
Lost 1 or more stage*	0%	0%	0%	0%	1%	2%

*from those students who could gain in each scale

A larger percentage of students in decile 10 year 8 classes from the school that had been in the project in the previous year were at the top levels of the scales initially than were students from all schools that were in the project for the first year. A larger proportion of students new to the project stayed on the same stage, but also a larger proportion of these new students who could gain did so. Both groups of students appeared to benefit from the project.

Table 3.16. Comparison of gains made by year 8 students from decile 4 schools in the project for the first and second years.

	Additive 1st year in 2002	Additive 2nd year in 2002	Multiplicative 1st year in 2002	Multiplicative 2nd year in 2002	Proportional 1st year in 2002	Proportional 2nd year in 2002
Number assessed twice	302	223	293	215	291	207
Ceiling both times*	37%	16%	18%	9%	6%	0%
Gained 0*	30%	37%	29%	35%	44%	43%
Gained 1 or more stage*	31%	25%	53%	32%	50%	49%
Lost 1 or more stage*	1%	22%	0%	24%	0%	8%

*from those students who could gain in each scale

The main difference here was in the percentage of students from the school in the project for the second year who started at the ceiling level for additive and multiplicative strategies. This difference was considerably smaller than the same difference for decile 10 schools. A higher percentage of the students in schools in the project for the first time gained at least one stage. The similarity in percentage of gains in proportional strategies may relate to the fact that in 2001 many schools were reported to have paid little attention to this scale.

The high percentage of students from the decile 4 school in the project for the second year who were assessed at lower stages than expected is unusual. The facilitator for this school suggested that it might have been due to use of the final scores for 2001 as the year 8 starting scores when there had been many changes in the scales, or differences in stringency in scoring. Decreases were reported from Stage 4 to 3, from Stage 6 to 5, and from 7 to 6. The students who were scored at a lower level were from several different classes.

Summary

The important steps for mathematical achievement are captured in the three strategy stages. For additive strategies, at this age, it is important that students who have not moved from counting-on to using a variety of part-whole strategies for adding do so. By the end of the year, 85% of year 7 students and 90% of year 8 students were using these strategies. The important step for multiplication is from using additive strategies to thinking of groups of numbers as single units, or thinking multiplicatively. By the end of the year, 55% of year 7 students and 69% of year 8 students were thinking multiplicatively. By the end of the year, 29% of year 7 students and 38% of year 8 students were able to use two or more multiplicative strategies, or use proportional reasoning. The figures in Table 3.17 give the proportion of students who were at the

top two stages for each scale on both occasions and who moved to part-whole strategies (Stages 5 or 6), to multiplicative strategies (Stages 6 or 7), or to proportional strategies (Stages 7 or 8).

Table 3.17. Percentages of students in years 7 and 8 moving to part-whole strategies for adding, to multiplicative strategies for multiplication problems, and to proportional strategies for proportional problems.

	Year 7	Year 8
Used part-whole strategies for addition on both assessments	66%	71%
Moved to part-whole strategies for addition during the year	19%	19%
Used multiplicative strategies for multiplication on both assessments	33%	44%
Moved from using counting or adding strategies to multiplicative strategies during the year	22%	25%
Used proportional strategies for multiplication on both assessments	15%	21%
Moved to using proportional strategies during the year	14%	17%

Chapter 4 – Results of a Test of Pre-algebraic Manipulation in Arithmetic

A test of six types of pre-algebraic manipulation in arithmetic was given to 837 year 8 students from four schools. Two of these schools were involved in the Numeracy Project and two were not. This was an initial attempt to see if intermediate school students in the Numeracy Project performed differently from intermediate school students who were not in the project.

Nature of the Test

The test consisted of six sections, each of which involved the application of a principle that has an algebraic base and that should make solving numerical problems easier. The test is in Appendix C.

Task A

This involved addition problems that were made simpler to solve by adding a number to one addend and subtracting the same number from the other addend in compensation. For example, 47 + 55 is easier to do mentally if 3 is added to 47 and taken away from 55, making the problem 50 + 52. Algebraically, $a + b = (a + x) + (b – x)$.

Task B

This involved subtraction problems in which the same number was added to each of the initial numbers to make the problem easier to work out. For example, 87 – 48 is the same as 89 – 50. Algebraically, $a – b = (a + x) – (b + x)$.

Task C

This involves applying the distributive law to multiplication. For example, 3 x 88 is the same as 3 x 90 – 6. Algebraically, $a(b – c) = ab – ac$.

Task D

This involves knowing what operation to use to complete an addition or subtraction statement with one number missing, a basic aspect of equivalence. Essentially, the student needs to know whether to add or to subtract the given numbers to fill in the box in a problem like □ + 26 = 431.

Task E

Solving a multiplication problem by multiplying one number and dividing the other by the same number. Thus 5 x 18 is the same as 5 x 2 x 18/2 (multiplying by 2/2, which equals one). Algebraically, a x b = ay x b/y.

Task F

Proportional reasoning in finding equivalent fractions. For example, 3/4 = 15/□. Algebraically, this is multiplying each fraction by an equivalent of one, in this case, 5/5.

All of these processes lead to actions that students traditionally learn first in algebra, for example when they "cancel out", factorise, or add the same number to each side of an equation. However, students involved in the Numeracy Project have the opportunity to use these techniques in solving additive, multiplicative, and proportional problems. Their understanding of these concepts in arithmetical operations should simplify the introduction of the same concepts in algebra.

The accepted answers, algebraic rationale, and analysis of variance for this test appear in Appendix D.

Participating Schools

Four intermediate schools were chosen to participate in this assessment. Two were participating in the project and two were not.

School A was using the Numeracy Project (NP). It was in a city of about 50 000. It was rated as a medium decile school. This was in the first year in the project.

School B was not in the Numeracy Project. It was also a medium decile school and was in the same city as School A.

School C was a high decile school in the second year of involvement in the Numeracy Project (NP). It is in a city of 1.2 million.

School D was not in the Numeracy Project. It was also a high decile school and was in the same large city as School C.

Scoring

Items were scored as correct on Tasks A, B, C, and E if students showed evidence of using the strategy modelled in the test. In order to be scored as using the principle, they had to choose a number to compensate, add, or multiply by, that made the problem easier to do mentally. Tasks D and F required only the correct answer, or evidence that a correct procedure was used even if there was a minor calculation error.

Results and Discussion

Table 4.1 gives the percentages of students who applied the required strategy in solving each group of problems.

Table 4.1. Percentages of students in each school who applied the strategy required for each set of items.

School	Decile	A	B	C	D	E	F	No. of students
A (NP)	Medium	74%	42%	32%	56%	6%	29%	159
B	Medium	60%	37%	28%	64%	1%	27%	244
C (NP)	High	84%	66%	65%	86%	7%	59%	210
D	High	72%	61%	38%	63%	2%	23%	224

Note that the majority of students, whether in the Numeracy Project or not, were successful on Task A, applying compensation to addition, and Task D, equivalence in addition and subtraction.

An analysis of variance was performed on the total score that each student obtained. It showed that the two upper decile schools (C and D) scored significantly higher than the lower decile schools, that schools in the Numeracy Project scored significantly higher than schools not in the Numeracy Project, and that there was an interaction between upper and lower decile and involvement in the Numeracy Project. All levels of significance were <0.01. (See Appendix D.) The decile 10 school that was in its second year of involvement in the Numeracy Project for two years significantly outscored all other schools.

Table 4.2. Mean number of items correct, and standard deviation for each school

School	Mean	Std. Dev.	No. of Students
School A (NP)	8.00	5.56	244
School B	7.44	4.81	159
School C (NP)	12.39	4.82	210
School D	8.32	4.91	224

These means are graphed in Figure 4.1.

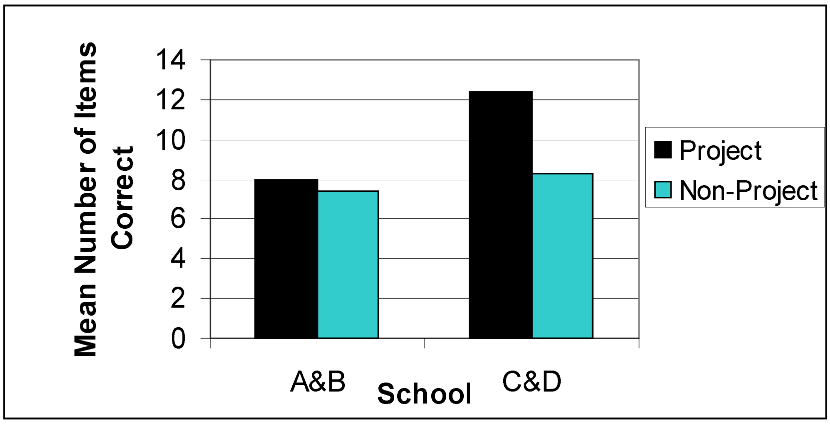


Figure 4.1. Mean number of items on which students were correct in project and non-project schools, in two areas of New Zealand, using the total number of correct answers regardless of time allowed for testing.

These results demonstrate that students from schools in the project were better able to apply the principles involved to than were students from similar schools that were not in the project. Much of this effect was due to the inclusion of a high decile intermediate school that was in the project for a second year. However, the other high decile intermediate school had specialist teachers for all mathematics classes and gave students four extra periods of numeracy per week for half of the year. Thus, this was a comparison of two schools that attended to numeracy, but had different special programmes to address students' knowledge in this sphere.

Students appeared to understand the instructions for all sections except Task E. The low percentage of students succeeding on this subtest may have been because they thought that a different technique was required, rather than finding numbers to divide and multiply by that made the problem simpler. Many students did this section either by multiplying and then dividing the answer by the given multiplicand or by first multiplying and then dividing by nine or 10 as in the sample.

Summary

The main findings of this study were that:

1. The students in the Numeracy Project did significantly better than the students in schools not in the project on this test.

2. The majority of year 8 students in all schools were able to apply the principle of compensation to addition problems and chose reasonable numbers to use in order to make the numbers easier to work with.

3. The majority of students in all schools correctly completed addition and subtraction problems with missing addends or subtrahends. The most difficult item of these four was $\square - 34 = 21$. Students who erred here usually subtracted rather than added the numbers given. This may have been because all other items required finding the difference between two numbers and this one required addition.

4. Very few students from any school used the strategy of multiplying and dividing by the same number for Task E. As said above, this could have been related to unclear directions. However, this may also be a difficult concept.

5. The two higher decile schools did better than the two lower decile schools, with students in the high decile school in the project for a second year out-performing all other schools.

   These results strongly suggest that the Numeracy Project is providing students with strategies that will transfer to algebra, making algebra a more meaningful topic for them.

Chapter 5 – Performance of Students in Years 9 and 10

Decile Levels of the Students' Schools

There were 1,446 year 9 students and 289 year 10 students from schools that participated in the Numeracy Exploratory study in 2002. The year 9 students spread across the range of deciles, with most students (76%) in the lowest four deciles. Most of the year 10 students came from one decile 1 school. In that decile 1 school, results for the top stream classes were not returned. It would be inappropriate to think of results for year 10 students as representative of that year group.

Table 5.1. Distribution by decile of year 9 and 10 schools and students.

Decile	Number of schools		Number of students		Percentage of students	
	year 9	year 10	year 9	year 10	year 9	year 10
1	2	2	206	199	14%	69%
2	2	0	158	0	11%	0%
3	3	0	462	0	32%	0%
4	3	0	269	0	19%	0%
5	0	2	0	0	0%	0%
6	1	1	87	83	6%	29%
7	0	0	0	0	0%	0%
8	2	1	173	7	12%	2%
9	1	0	91	0	6%	0%
10	0	0	0	0	0%	0%
Totals	14	6	1446	289	100%	100%

Only three of these schools were new to the project in 2002. These were a small decile 2 school, a decile 6 school, and a decile 8 school. Two of the schools that participated in 2001 did not enter final data in 2001 but did so in 2002. They were considered to be in the project for the second year although their 2001 data are not available for comparison.

Gender and Ethnicity of Year 9 and 10 Students

Participating were 776 girls and 670 boys from year 9 and 249 girls and 40 boys from year 10. Two of the schools submitting results for year 10 schools were girls' schools. Overall, ten schools were co-educational, three schools were single sex girls' schools (decile 1, 4, and 6) and one school was a single sex boys' school (decile 8). See Figure 5.2.

Table 5.2. Gender of year 9 and 10 students.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Girls	776	54%	249	86%
Boys	670	46%	40	14%

Table 5.3. Ethnicities of year 9 and 10 students in the project in 2002.

	Year 9		Year 10	
European	745	52%	52	18%
Māori	345	24%	14	5%
Pasifika	257	18%	200	69%
Asian	41	3%	8	3%
Other	58	4%	14	5%
Total	1446	100%	289	100%

The distribution of students by ethnicity differed with the decile of the school. This was more marked for schools that were in the project for the first time in 2002 because of the characteristics of the schools included. Table 5.3 gives the distribution, by ethnicity, for year 9 and 10 students.

Table 5.4. Ethnicity of students by decile ranking.

	Decile 1–3	Decile 4–6	Decile 8–9	Decile 1	Decile 6	Decile 8
	n=826	n=356	n=264	n=199	n=83	n=7
Year	9	9	9	10	10	10
European	34%	67%	86%	1%	54%	86%
Māori	32%	19%	5%	5%	5%	14%
Pasifika	28%	6%	2%	91%	22%	0%
Asian	2%	4%	2%	2%	7%	0%
Other	4%	4%	4%	2%	12%	0%

Thus the majority of students in the lower three deciles were of Māori or Pasifika ethnicity. In the year 10 sample from the decile 1 school, 91% of the students were of Pasifika descent. In the middle decile schools over half of the students were European. In the upper decile schools 86% of the students were European.

There was also a difference in distribution of ethnicity in the project for the second year and the first year in 2002. This is given in Table 5.5.

Table 5.5. Distribution of ethnicity by decile for schools in the project for the second and first year for year 9.

Group	Total in project		Decile 1–3		Decile 4–6		Decile 8–9	
Year in	1st	2nd	1st	2nd	1st	2nd	1st	2nd
project	n=134	n=1446	n=11	n=1312	n=88	n=268	n=34	n=229
European	64%	50%	0%	34%	60%	69%	97%	85%
Māori	11%	25%	100%	31%	3%	24%	%	6%
Pasifika	11%	18%	0%	29%	17%	2%	%	2%
Asian	7%	2%	0%	2%	11%	2%	%	3%
Other	6%	4%	0%	4%	17%	3%	3%	4%

Lower decile schools that were in the project for both years had a higher proportion of students of Māori or Pasifika ethnicity than did the lower decile schools that were new to the project in 2002. Upper decile schools in the project for two years had a higher proportion of European students than did those schools that were new in 2002. This differentiation is more marked in the few schools that entered the project in 2002 because these schools were strongly weighted by ethnicity.

Because ethnicity and gender varied with decile ranking, results have not been analysed separately by either of these factors.

Very few students were listed as having been in the project in the previous year, so this factor was not analysed separately.

Performance on Strategy Tasks

See Appendix E for the percentages of year 9 and 10 students at each stage on each scale.

Additive strategies

The percentages of year 9 and 10 students at each stage are presented in Appendix E.

Results for year 9 and 10 are presented separately. The year 9 schools were spread more evenly across deciles and ethnicity, as reported above, but the year 10 classes that returned results appeared to have been only from students who were judged as being in need of extra help in numeracy.

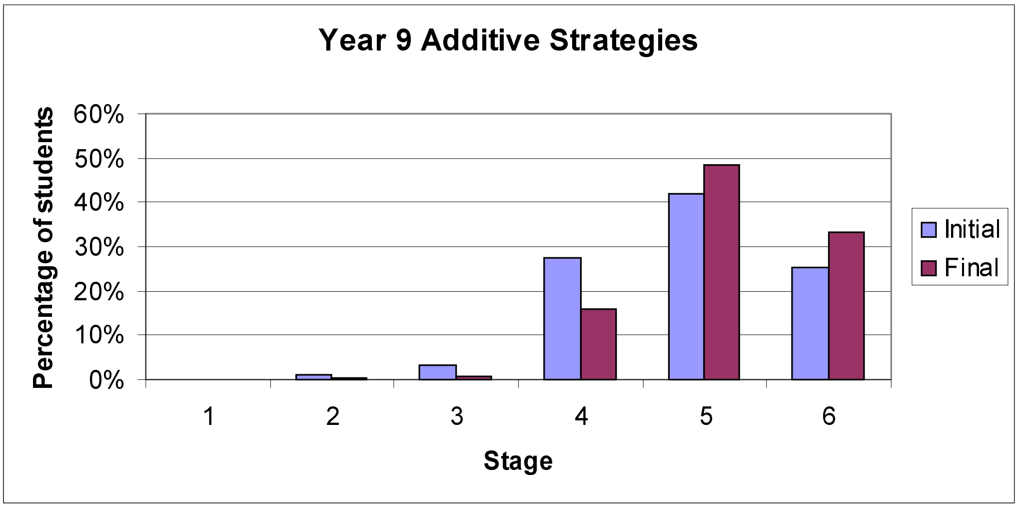


Figure 5.1. Percentages of year 9 students at each stage on additive strategies.

Nearly all year 9 students were asked these questions. Small percentages were seen to be at Stage 2 or 3, needing to add by counting all items, starting with one. Initially, 27% of students preferred to add by counting on from the larger number rather than using more advanced strategies. This decreased to 16% after students had practice in using part-whole strategies.

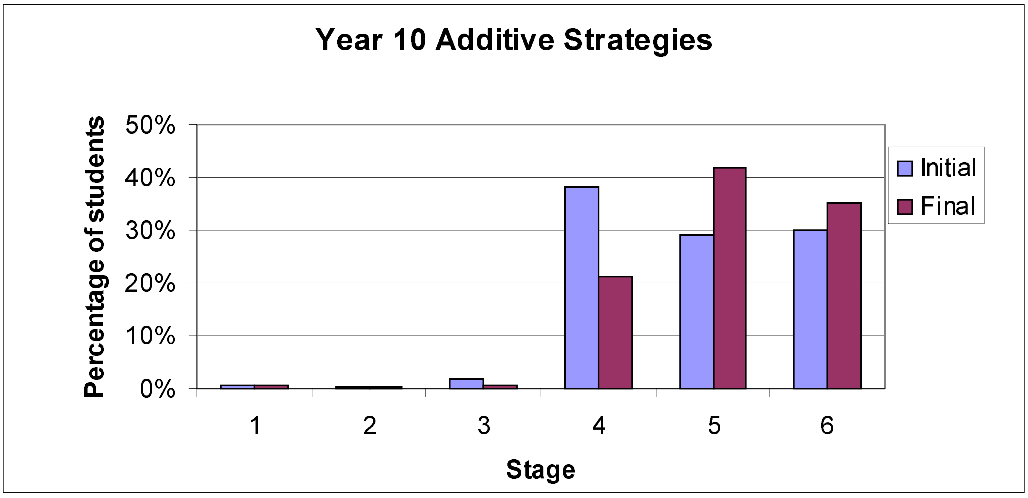


Figure 5.2. Additive strategies of year 10 students.

The year 10 classes included in the project were new to the project and appeared to be mainly those in need of remedial help. The major shift for this group was from a relatively high percentage of students adding by counting on, to a greater percentage adding with the use of part-whole additive strategies, presumably because of instruction. Fifty-three students, or 73% of the 71 students who gained one or more stages, moved to part-whole thinking.

Gain in additive strategies

Table 5.6 gives the numbers and percentages of students changing stages between the initial and final assessments. Three hundred and eighty-seven students, or 37%, gained at least one stage. The percentage of students using either early or advanced part-whole strategies for adding increased from 67% to 81% during the project. Two hundred and twenty-three students, or 21%, made the move to additive thinking, either early or advanced.

Table 5.6. Summary of progress made by year 9 and 10 students on assessment of additive strategies.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	32	2%	5	2%
Ceiling*	488	35%	85	29%
Gain 0 if could gain*	522	37%	129	45%
Gained 1 or more stage*	375	27%	71	25%
Lost 1 or more stage*	29	2%	4	1%

*from those students assessed on both occasions (N for year 9 = 1,382, N for year 10 = 289)

Multiplicative strategies

Forty percent of year 9 students were using multiplicative thinking (Stage 6 or 7) for these problems when first assessed. On the second assessment, 56% of year 9 students were credited with using multiplicative thinking. The others either were not given this assessment, used counting, or used adding to solve multiplication problems.

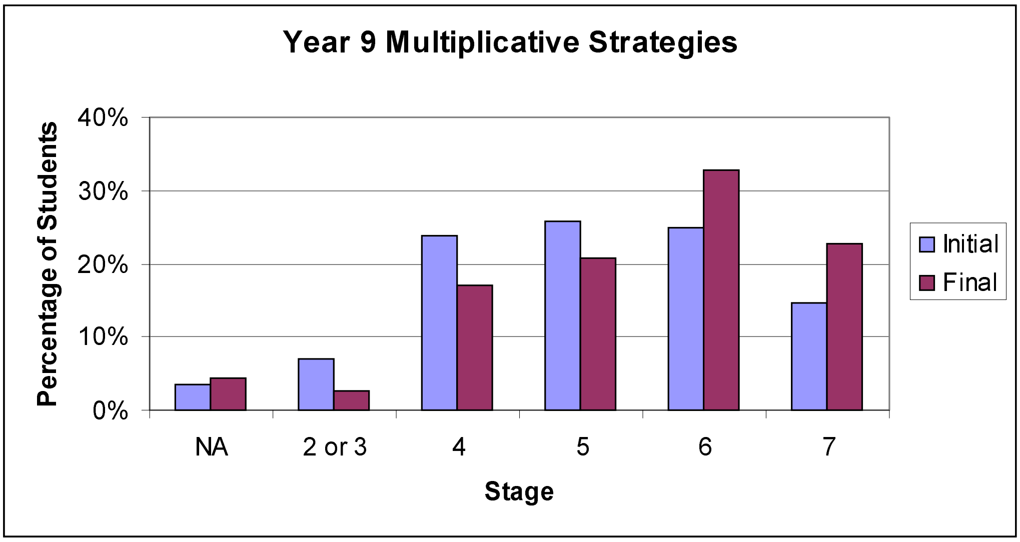


Figure 5.3. Percentages of year 9 students at each stage on multiplicative strategies.

Of the year 10 students assessed, 30% used multiplication strategies initially and 48% used these procedures on the second occasion. The others, if assessed, used counting or adding to solve multiplication problems.

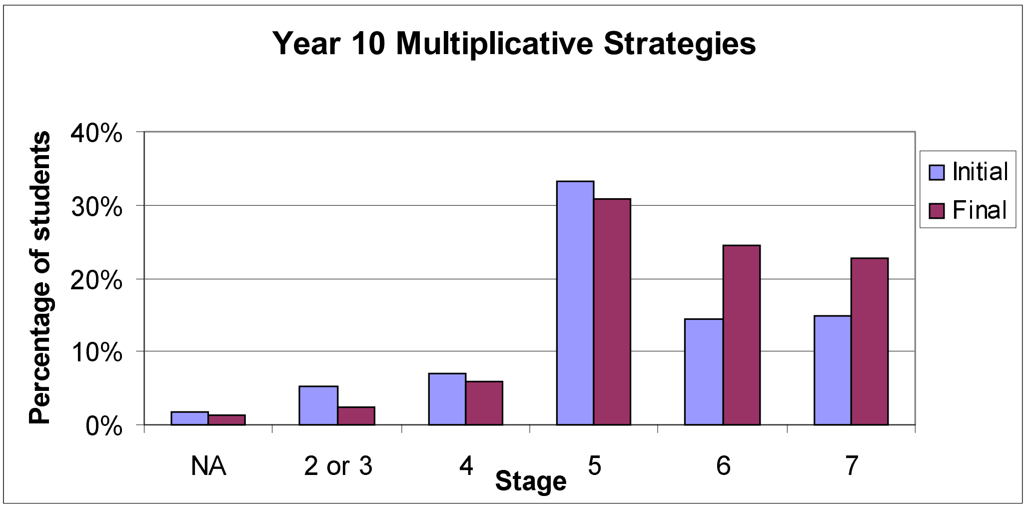


Figure 5.4. Percentages of year 10 students on each stage for multiplicative strategies.

Table 5.7 gives the numbers and percentages of students assessed who did or did not change their strategies for these problems.

Table 5.7. Summary of progress made by year 9 and 10 on assessment of multiplicative strategies

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	91	6%	5	2%
Ceiling*	202	15%	42	15%
Gain 0 if could gain*	611	45%	136	48%
Gained 1 or more stage*	509	38%	103	36%
Lost 1 or more stage*	33	2%	3	1%

*from those students assessed on both occasions (N for year 9 = 1,355, N for year 10 = 284)

Of those students in year 9 who were tested on both occasions and were not already at ceiling, 45% gained at least one stage. This finding is similar to that of 2001 (Irwin and Niederer, 2002).

The important move on this scale is from doing multiplication using some adding procedure, as signified in Stages 2, 3, 4, and 5 and using a multiplicative procedure as indicated in Stages 6 and 7. Among the year 9 students, 216 students (47%) made this important step. Of the year 10 students, 50 (42%) moved to using part-whole strategies.

Proportional reasoning

Students at the top three stages on this scale were considered to be using multiplicative strategies, while those at Stages 7 and 8 were using early and advanced proportional strategies.

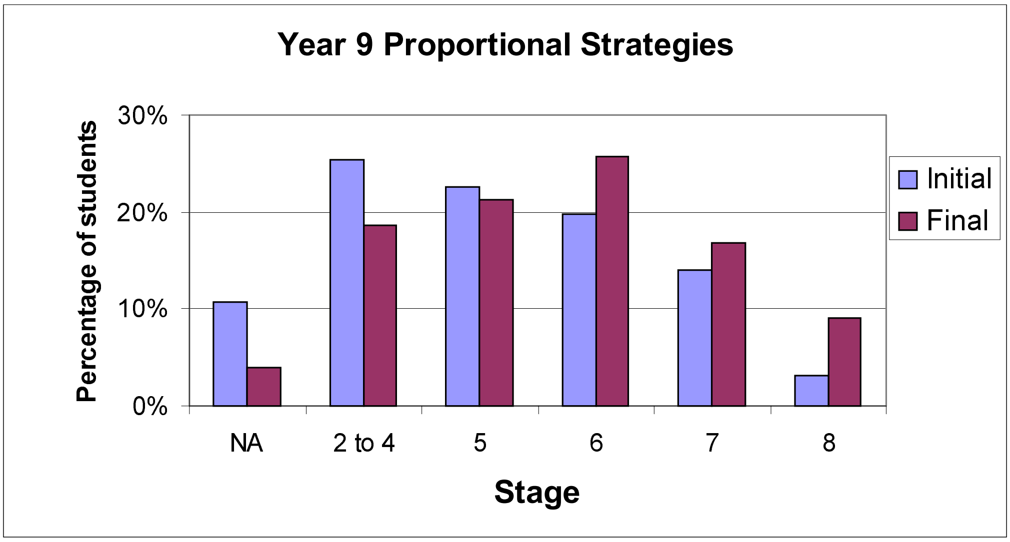


Figure 5.5. Percentages of year 9 at each stage for proportional strategies.

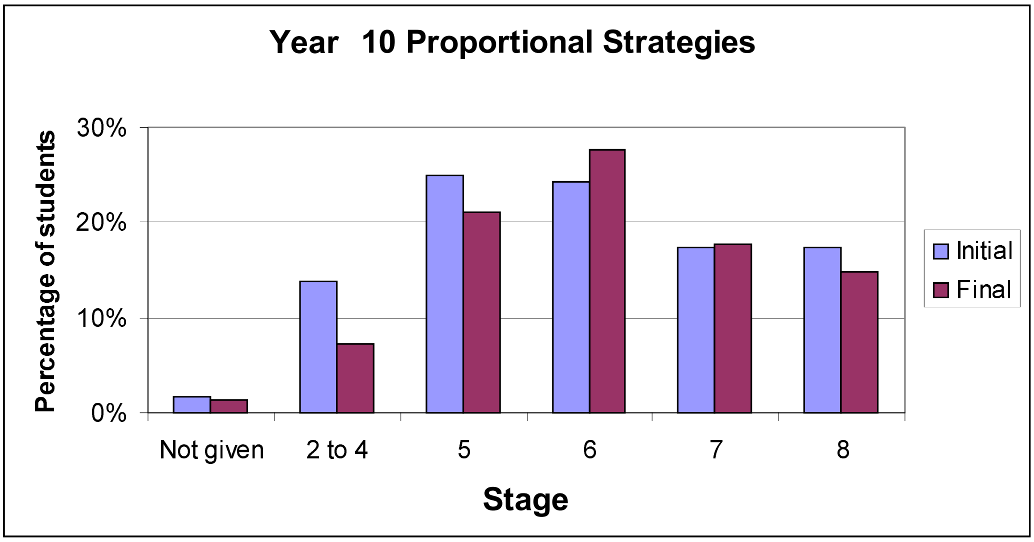


Figure 5.6. Percentages of year 10 students at each stage for proportional strategies.

Of the year 9 students, 132 students, or 10%, moved to proportional thinking, the ability to use more than one multiplicative process to solve a problem. In year 10, 21 students, or 7%, moved to proportional thinking. The lower percentage of year 10 students may be due to the fact that the skill represented in this scale is usually taught only after additive and multiplicative strategies have been addressed. It may also be related to the nature of the sample for that year.

Table 5.8. Summary of progress made by year 9 and 10 students on assessment of proportional strategies.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	106	7%	5	32%
Ceiling*	45	3%	2	1%
Gain 0 if could gain*	686	51%	176	62%
Gained 1 or more stage*	574	43%	104	37%
Lost 1 or more stage*	35	3%	2	1%

*From those students assessed on both occasions (N for year 9 = 1,340, N for year 10 = 284)

Simpler Knowledge Scales for Whole Number Identification, Forward Number Word Sequence, and Backward Number Word Sequence

Relatively small proportions of year 9 and 10 students were assessed on these scales. Percentages at each stage (initial and final) are given in the following table.

Table 5.9. Percentages of year 9 and 10 students at each stage on scales of number identification, forward number word sequence, and backward number word sequence on initial and final assessment.

	Whole Number Identification		FWNS		BWNS		Whole Number Identification		FWNS		BWNS	
Year	9		9		9		10		10		10	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Not given	90%	91%					57%	59%				
0	1%	0%	1%	3%	1%	1%	0%	0%	0%	0%	0%	0%
1	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
2	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3	0%	0%	1%	1%	1%	1%	3%	2%	2%	0%	0%	0%
4	9%	8%	12%	6%	15%	8%	40%	39%	10%	3%	13%	4%
5			46%	33%	43%	31%			48%	27%	45%	24%
6			40%	58%	39%	59%			41%	70%	42%	72%

On these relatively simple knowledge scales, the vast majority of year 9 students were not given the test of whole number identification, which appears only on Form A. Most of those who were assessed on this scale were at the top stage. Thus they could read numbers in the hundreds. A higher percentage of year 10 students were assessed on this test. However those who were assessed on it were also able to read numbers in the 100s.

Assessment of forward and backward number sequences were on Forms B and C and given to the majority of year 9 and 10 students. Some gain was shown by students in both years. By the final assessment, there were still about 40% of the year 9 students and about 30% of students in year 10 who could not give the number before and after a six-digit numeral.

More Complex Knowledge Scales

As indicated in Task 2 of this report on scale difficulty, the tests of knowledge of fractions, decimals and percentages, and grouping and place-value were more difficult. In viewing the percentage of students at each stage initially and finally, the difference in the assessment form should be borne in mind.

Knowledge of fractions

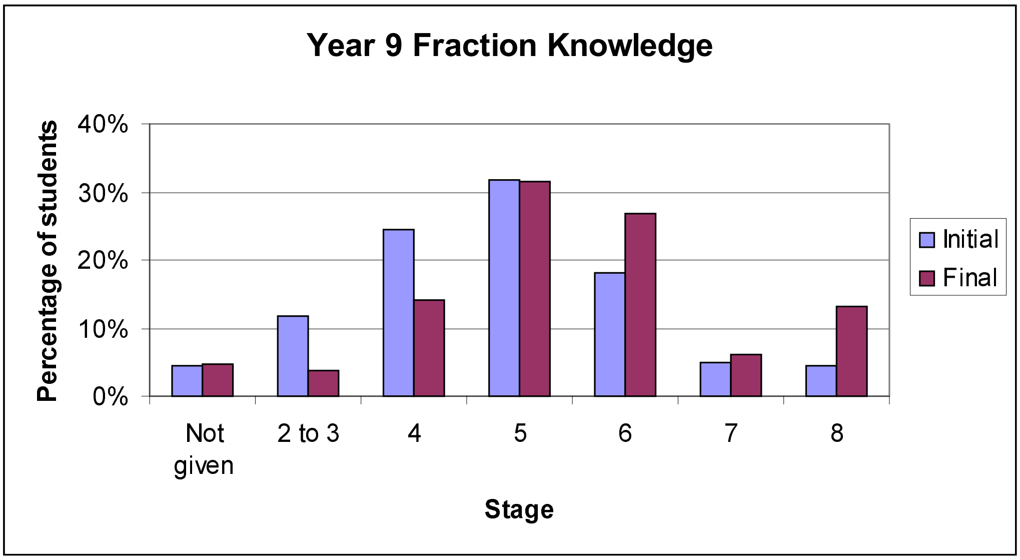


Figure 5.7. Percentages of year 9 students at each stage in knowledge of fractions.

Most students, both initially and finally, were at Stage 5, being able to order unit fractions. On initial assessment, 36% of the students were at Stages 2–4, either not understanding the regional, or area, model of fractions or not being able to order them. By the final assessment, this proportion had decreased to 18%. On final assessment, 46% of year 9 students could coordinate numerators and denominators, give equivalent fractions, and order fractions with different numerators and denominators. As students assessed on Form B could only score up to Stage 6, we cannot tell whether some of those assessed as being at this stage in the final assessment might have scored more highly if given Form C.

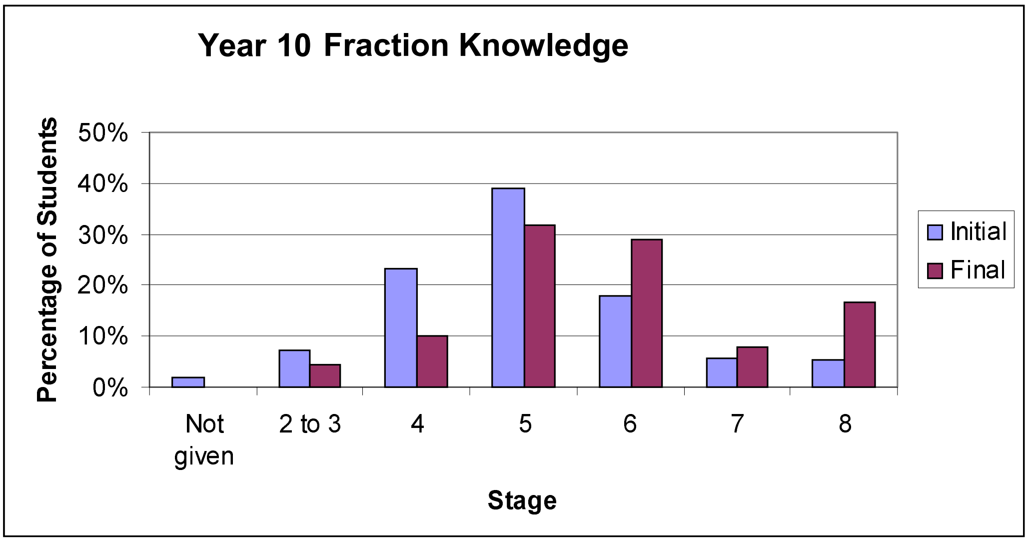


Figure 5.8. Percentages of year 10 students at each stage in knowledge of fractions.

As with the year 9 students, the largest percentage of students on both occasions was at Stage 5, able to read unit fractions. On initial assessment, 30% could not do this, and on final assessment, 14% could not do this. Initially, 29% were at the three top stages, and on the final assessment 54% were above these stages. As with the year 9

students, it is not possible to tell how many more students might have been at the top two stages had they been given Form C.

Table 5.10. Summary of progress made by year 9 and 10 students on assessment of knowledge of fractions.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	56	4%	3	1%
Ceiling*	519	37%	14	5%
Gain 0 if could gain*	409	29%	126	44%
Gained 1 or more stage*	404	29%	140	49%
Lost 1 or more stage*	58	4%	6	2%

*from those students assessed on both occasions (N for year 9 = 1,390, N for year 10 = 286)

A higher percentage of the year 9 than of the year 10 students was at the top level initially.

Knowledge of decimals and percentages

This subtest appeared only on Form C. The scale starts at Stage 4, yet some students were assessed as being at Stages 2–3. As these stages have no precise meaning, the number and percentage for those so assessed has been included with the category "not given". As a minority of students from either year were assessed on this scale, results are presented in a table.

Table 5.11. Numbers and percentages of year 9 and 10 students either not assessed or assessed as at each stage on testing knowledge of decimal and percents.

	Year 9		Year 10	
	Initial	Final	Initial	Final
Not given	70%	60%	66%	33%
Stage 4	4%	2%	4%	6%
Stage 5	10%	11%	8%	16%
Stage 6	6%	10%	7%	12%
Stage 7	5%	5%	6%	13%
Stage 8	4%	12%	9%	20%

Gains in knowledge of decimals and percentages

This scale was not given on either assessment to 75% of the year 9 students and 70% of the year 10 students. Giving it on the second occasion but not on the first (a common occurrence) did not allow gain to be assessed. The figures presented below are related to the small percentage of students in each year given this scale on both occasions.

Table 5.12. Summary of progress made by year 9 and 10 students assessed on knowledge of decimals and percentages.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	1084	75%	201	70%
Ceiling*	57	16%	26	30%
Gain 0 if could gain*	128	35%	12	14%
Gained 1 or more stage*	164	45%	50	57%
Lost 1 or more stage*	13	4%	0	0 %

*from those students assessed on both occasions (N for year 9 = 362, N for year 10 = 88)

The percentages of students gaining (where the data was available) for year 9 and year 10 students were 45% and 57% respectively.

Gains in knowledge of grouping and place-value

On Form A, this scale involves recognising groups of five or ten dots and knowing how many would be in several groups of these groups. On Forms B and C all items involved knowing how many 10s, 100s, tenths, or hundredths would be in a given numeral. Hence, at these stages, it is a test of understanding the full meaning of place-value, not the more limited use of this term to imply place-value of each numeral. The scale is represented on all three forms of the test although not all stages are on all forms.

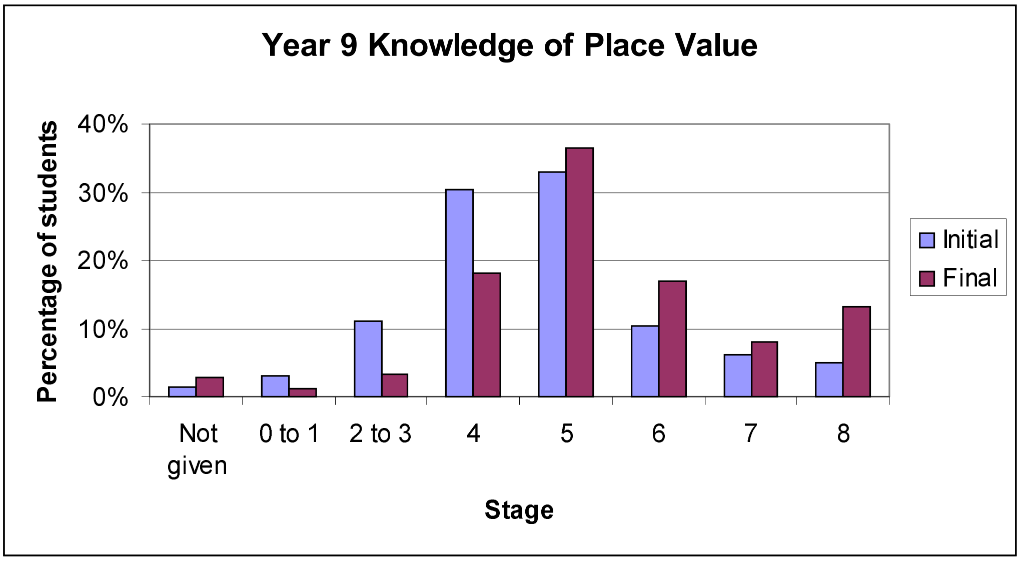


Figure 5.9. Percentages of year 9 students at each stage of knowledge of place-value.

Stage 5, knowledge of the number of 10s in numbers in the hundreds, was the most common stage reached by year 9 students, both initially and finally. Initially, 45% of students were at stages below this, and on the final assessment 25% of students were at these lower stages. Initially, 21% of students were at more advanced stages and 38% were at more advanced stages on the final analysis.

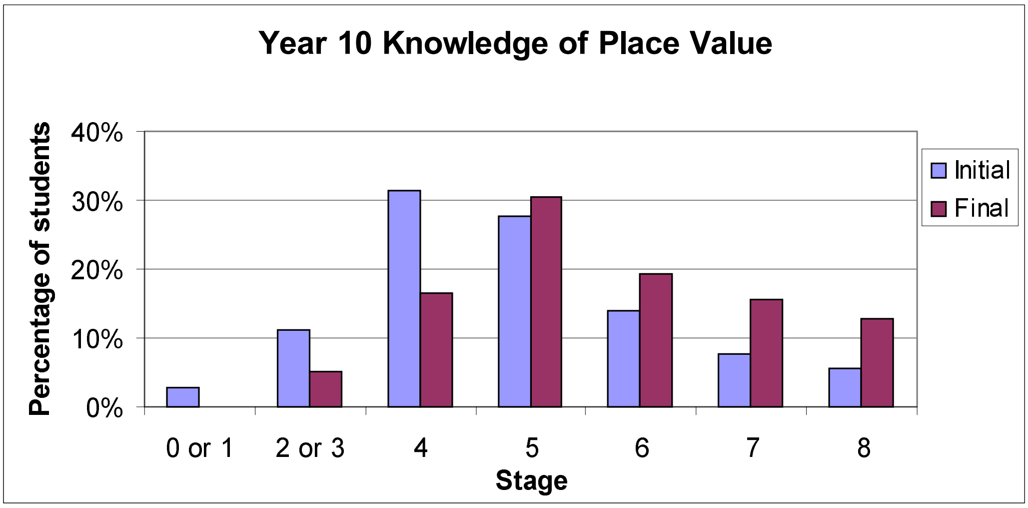


Figure 5.10. Percentages of year 10 students at each stage of knowledge of place-value.

The largest percentage of this unusual sample of year 10 students was initially at Stage 4, the ability to give the number of 10s in a two-digit number. On the second assessment, the largest group were able to give the number of 10s in three-digit number. The percentage of year 10 students at the top three stages of this scale, giving the number of 10s or 100s in any whole number and the number of tenths or hundredths in mixed numbers, increased from 28% initially to 48% on the final assessment.

Gains in knowledge of place-value

Increases in stages show good gains for both years. The greater gain for year 10 students may be related to their relatively lower starting point.

Table 5.13. Summary of progress made by year 9 and 10 students in knowledge of place-value.

	Year 9 Number	Year 9 Percentage	Year 10 Number	Year 10 Percentage
Not assessed twice	53	4%	2	1%
Ceiling*	67	5%	15	5%
Gain 0 if could gain*	576	42%	101	35%
Gained 1 or more stage*	678	49%	168	59%
Lost 1 or more stage*	53	4%	3	1%

*from those students assessed on both occasions (N for year 9 = 1,374, N for year 10 = 287)

One reason for the large percentage gaining on this scale could be that students previously believed that only the place gave the value, that is, there would be no tens (or $10 notes) in $609. What they would have learned is that all places to the left of that specified are included in the number of 10s in such a number.

Summary

Gains can be meaningfully shown for the year 9 sample on four scales. These are given in Table 5.14.

Table 5.14. Percentages of students in year 9 and 10 either at ceiling or making progress on additive, multiplicative, and proportional strategies and on knowledge of place-value.

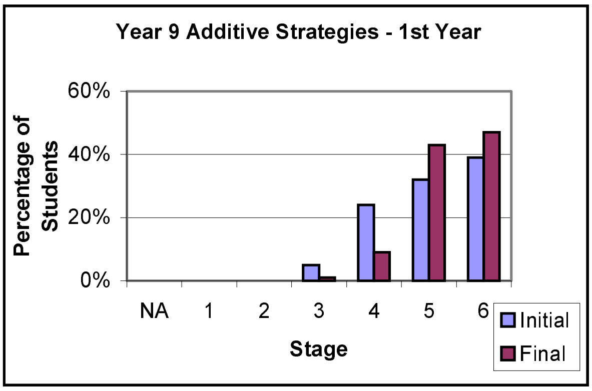
	At ceiling on both assessments	Gained 1 or more stage
Additive strategies	35%	27%
Multiplicative strategies	15%	38%
Proportional strategies	3%	43%
Knowledge of place-value	5%	49%

The percentage of students gaining at least one stage was similar to year 9 findings for 2001 (see Irwin and Niederer, 2002) for proportional strategies and place-value. The percentage of students gaining at least one stage on additive and multiplicative strategies is lower than that found for year 9 students in 2001 (45% and 44%, respectively, in 2001 in comparison with 27% and 38% in 2002). This may have been due to different procedures in recording the data, to what one facilitator has referred to as "mopping up" the deficiencies found previously, to difference in the sample, or to a different emphasis in teaching.

Comparison of Strategies for Schools in the First and Second Years of the Project

The data for the three strategy scales were compared for schools in the first and second year of the project. Although the students may have been new to the project, a large number of the teachers were in their second year and had experience with the project.

Additive strategies



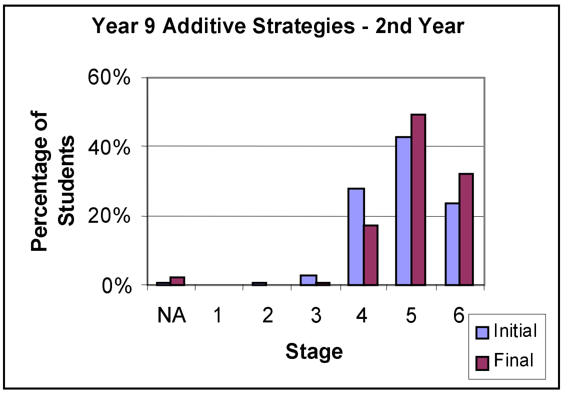
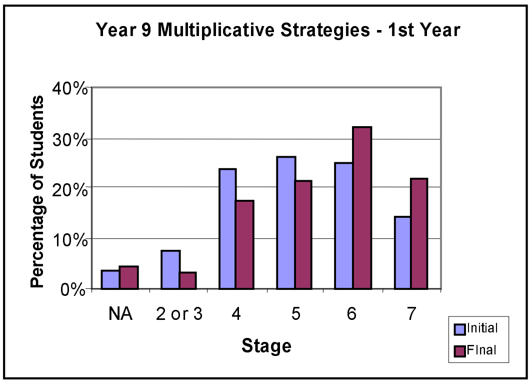


Figure 5.11. Additive strategies of year 9 students in schools in the project for the first and in the second (N for first year in project = 134, N for second year in project = 1,446).

In interpreting these graphs, it should be remembered that there were very few schools in the project for the first time in 2002. In the schools that were new to the project, 26 of 39 students (67%) who were initially using counting strategies moved to using a part-whole strategy for adding. For the schools that were in their second year in the project in 2002, 197 of 432 students (46%) moved from using a counting strategy to using a part-whole strategy for adding. This is similar to the finding for intermediate

students in the project for the second year, where more students were at the top levels initially, so fewer were able to gain.

Multiplicative strategies



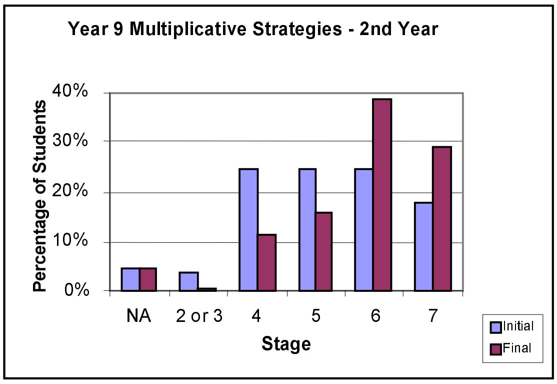
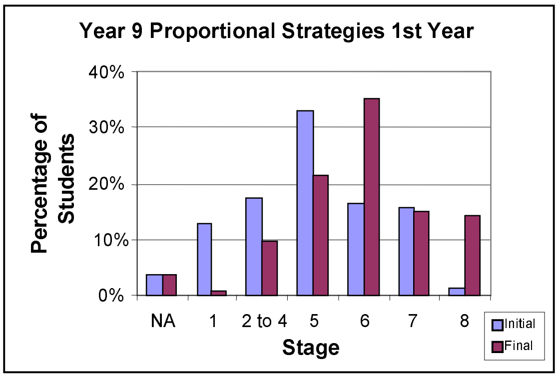


Figure 5.12. Percentages of students at multiplicative stages from schools who were in the project for the first and second years, initially and finally (N for first year in project = 134, N for second year in project = 1,446).

For schools in the second year in the project, 197 students (26%) made the important move to using strategies that were multiplicative rather than additive for these problems. Of year 9 students in schools new to the project, 35 of 71 students (49%) made the move to multiplicative thinking.

Proportional strategies



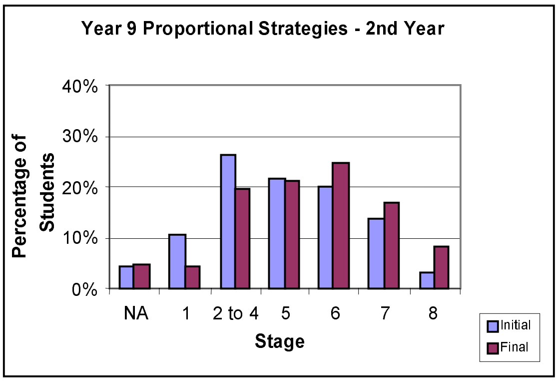


Figure 5.13. Percentages of students using different proportional strategies who were in the project for the first or second year (N for first year in project = 134, N for second year in project = 1,446).

The important move on this scale is from below Stage 7, early proportional thinking, to Stage 7 or 8, proportional reasoning that involves using two or more multiplicative processes. Of students from schools in the project for the first year, 21 of 106 students (20%) made the move to proportional thinking from lower stages. Of students from schools in the project for the second year, 166 of 1030 (16%) moved to proportional thinking from lower stages.

Summary

Comparisons were made in gains in strategy over the important steps for addition (to part-whole), multiplication (to multiplicative thinking), and proportional tasks (to proportional thinking). Somewhat smaller percentages of students from schools with a year of experience already in the project made the main gains in strategy than did in schools who were new to the project. This may be related to the particular schools, especially in the new cohort. It may also be related to a statement made by one head of mathematics in the project for the second year. She reported that the teachers were somewhat less enthusiastic about the project in the second year and that it received less attention in the face of other innovations in the school mathematics programme.

Chapter 6 – Comparison of the Multiplicative Skills of Year 9 Students from Upper and Lower Decile Schools

Gelman (1999) suggests that multiplicative concepts are not among the naïve mathematical concepts everyone learns. Tirosh and Graeber (1990) and others describe the difficulties that pre-service teachers have with mathematical concepts that involve multiplicative thinking. Yet many activities, including operating with rates and fractions, require the flexible use of multiplicative procedures. The development of multiplicative strategies is probably the most important section of the Numeracy Project for the age range that is the subject of this report. If students have not developed multiplicative thinking by year 9, it could be the most important focus of a remedial programme.

A multiplicative concept is defined as any concept that requires considering groups of numbers as a single unit. Piaget (1985/1987) discusses multiplication as being more complex than addition, as it involves implicit quantification. Students who operate multiplicatively know that there is a certain quantity in each of the numbers multiplied, but do not need to refer to the individual items or numbers in a group. He describes several stages that young children go through as they develop this understanding, with Levels IIB and III being truly multiplicative.

Mulligan and Mitchelmore (1997) also present a developmental model for young children's approaches to multiplication problems. Their model shows multiplicative concepts as arising out of additive ones. The developmental pattern that these Australian authors describe is similar to that used in the project and is similar to that commonly used in New Zealand schools. In this model, children move from direct counting to rhythmic counting, skip counting, additive calculation, and finally to multiplicative calculation.

Both Mulligan and Mitchelmore and Piaget describe the nature of multiplicative thinking used by young children, aged 7–10. Yet, as indicated above, many adults and older students fail to develop multiplicative thinking. Students continue to use additive calculation or repeated addition and do not move to multiplicative strategies. While using addition appropriately may give accurate answers, it is time consuming for more than the simplest problems and does not permit students to understand the more complex activities of finding a fraction of a number, or working with rates and ratios. These students fail to move to the level of implicit quantification that Piaget refers to as being seen in much younger children.

Schools Compared in This Analysis

This chapter compares the results of two decile 1 schools, one decile 8 school, and and one decile 9 school. There were 189 students from the two decile 1 schools and

225 students from the decile 8 and 9 schools who were assessed using the multiplicative strategies assessment on two occasions.

Students in the two decile 1 schools were 88% from a Pasifika background. Students from the upper decile schools were 85% of European background. As decile ranking was initially based upon correlation of factors such as ethnicity and passes in School Certificate (Dialogue Consultants, 1990), the fact that students in upper decile schools do better than students in lower decile schools is not surprising. However, it was hoped that the Numeracy Project would be of use as a remedial programme in lower decile schools. If it were a successful remedial programme, students from lower decile schools would make gains in progress that would not bring them to the level of students in upper decile schools.

The following table shows the initial and final percentages of year 9 students, from two lower and two upper decile schools, who used multiplicative strategies for both multiplication and proportional problems.

Table 6.1. Percentages of students who used multiplicative strategies on assessment of multiplication and proportional problems from lower and upper decile schools.

Decile 1 schools N=189		Decile 8–9 schools N =225	
Initial	Final	Initial	Final
24%	34%	66%	83%

Statistical analysis (Newcombe, 1998) showed that a significantly smaller proportion of students in decile 1 schools used multiplicative strategies, both at the start and finish, than did of students in the decile 8 or 9 schools ($p < .01$). An increased number of students from both groups came to use multiplicative strategies, but by following the recommendations of the project to teach the next higher stage in the usual developmental progression, students from lower decile schools had much less opportunity to become multiplicative thinkers because they started at lower stages.

In terms of the stages provided in the project, 40% of the students in decile 1 schools, not already at the ceiling level, improved and 49% of the students from the decile 8 and 9 schools, not already at ceiling, improved. Table 2 shows that their improvement was at different levels.

Table 6.2. Percentages of students gaining at least one stage on the Number Framework (based on the number of students not already at ceiling).

	Decile 1	Decile 8–9
Students on ceiling initially	5%	37%
Total students gaining at least one stage	40%	49%
Students gaining within additive strategies (stages to 5)	23%	7%
Students moving from additive to multiplicative strategies (≤ 5 to ≥ 6)	11%	25%
Students who gained within multiplicative strategies (Stages 6–8)	5%	17%

These data show that students did move up stages according to the hierarchy assumed by the project, a hierarchy also proposed by Piaget (1978) and by Mulligan and Mitchelmore (1997) for young children. However, adopting this progression left the students from lower economic areas still well behind their peers from more affluent

areas. In accordance with the directions of the project, most low decile students worked on more advanced additive strategies, whereas most upper decile students worked on multiplicative strategies. With this emphasis, it is not surprising that more than twice the percentage of upper decile students progressed from additive to multiplicative thinking.

Discussion

The main questions raised by these data are: (1) what brought about the increased use of multiplicative strategies, and (2) does this project, which emphasises methods used by much younger children, disadvantage lower decile students?

What brought about the change? Teachers reported that there had been major changes in their knowledge of individual students, and in their teaching. Teaching was different in each of the schools despite the suggestions from the project. Some teachers reported a change from their existing pattern of whole-class teaching, usually using a textbook, to teaching skills and strategies that they had not previously taught, and to teaching in groups. Others reported adding an initial portion to their lessons on number sense, working from their students' known levels. None of these schools abandoned their usual curriculum, but they did give more time to numeracy than previously. Comments made in 2001 and 2002 included:

They are finding the work within their means, so I can actually sit down with one or two or three students. It is that that is reaping the benefits. I am able to listen to them and hear what is going on in their heads and help them with the best strategy for them rather than doing one thing for the whole class.

Year 9 teacher in a lower decile school, 2001.

Most people would say that their classes are happier. That doesn't mean that they are more saintly but certainly they are happier because they have things that they can do. The kids in the bottom group are much happier. It has been most successful for them.

Head of a mathematics department in a lower decile school, 2001.

They are listening to their students, and moving from there.

Facilitator, 2002.

Listening to students has been seen as essential to good teaching from Plato through to current educators. Constructivist classes are characterised by teachers listening to students and students listening to one another (e.g., Kamii and Warrington, 1997). Yet these secondary teachers had possibly been preoccupied by their own teaching agenda and not had the time to listen to their students. The interviews gave them the initial opportunity to listen, and facilitators helped them to continue to listen while in the project.

Does the project continue to disadvantage lower socio-economic students by encouraging them to move up through a framework developed for young children? This is a serious concern, especially as one hope was that the experimental project would prove to be remedial for this group. However, in using a developmental framework appropriate for young children the project developers apparently expected

older children to move through the same stages. These students may have only two more years of schooling ahead of them and are unlikely to spend much more time on numerical concepts. This suggests that the majority will leave school as additive thinkers. It might be more appropriate to introduce them directly to thinking about groups of numbers as units, with inherent quantification. One head of mathematics from a decile 1 school commented that these students are overly dependent on algorithms.

We need to teach them to go back to skip counting. They see a hard multiplication problem and want to do it with the algorithm rather than seeing that they could multiply it by a larger number and subtract.

Project coordinator, decile 1 school, 2002.

Many teachers have commented that when elementary school students have been through the project, this problem will not be seen in secondary schools. It seems unlikely that this problem will go away that easily. It would seem more important to introduce these secondary school students directly to thinking of nested quantities, as in Piaget's Levels IIB and III (Piaget 1983/1987). This would be more in the spirit of remedial programs for adults such as that introduced by Triesman (Mathematics Department, University of Illinois, 2002). Engaging the students in the value and power of multiplicative thinking as young adults could be more beneficial than expecting them to move up through the stages of young children.

Chapter 7 – Comparison of Years 7, 8, and 9 on Strategy Scales

An interesting aspect of the 2001 results was that there was little difference between years 7, 8, and 9 on initial scale scores. Final results showed year 8 students to have made more gains than either year 7 or year 9.

Similar results were found in 2002. However, 54% of the year 7 and 8 students and 76% of the year 9 students came from schools in the bottom four deciles. The scale scores obtained may relate more to decile ranking than to other factors.

The following figures show the comparative stages of students in years 7, 8, and 9 on the strategy scales.

Comparison of Additive Stages

A small percentage of students were not assessed or assessed as being below Stage 4 for this scale. Most students were judged to be achieving at Stages 4, 5, or 6. Year 8 had a higher proportion of students who reached Stage 6, advanced part-whole, by the end of the project than did years 7 or 9. The largest percentage of year 9 students finished the project at Stage 5, early part-whole.

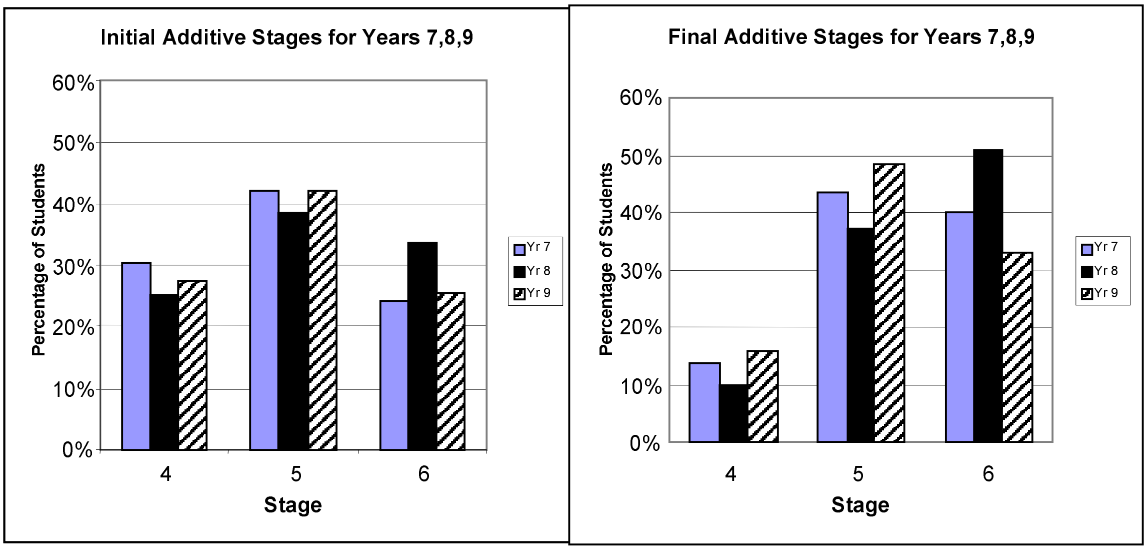
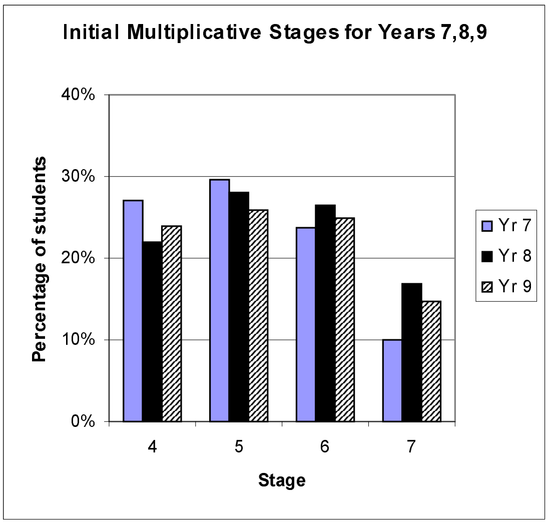


Figure 7.1. Initial and final stages on additive strategies for years 7, 8, and 9.

Comparison of Multiplicative Stages

Most students scored at Stages 4, 5, 6, and 7 on this scale. The percentages of those either not assessed on this scale or scoring at Stage 2 or 3 were: year 7 – 10% initially and 5% finally; year 8 – 7 % initially and 3% finally, year 9 – 11% initially and 7% finally.



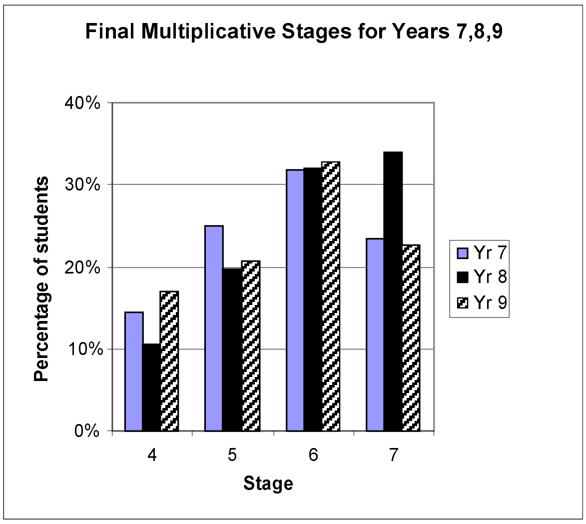
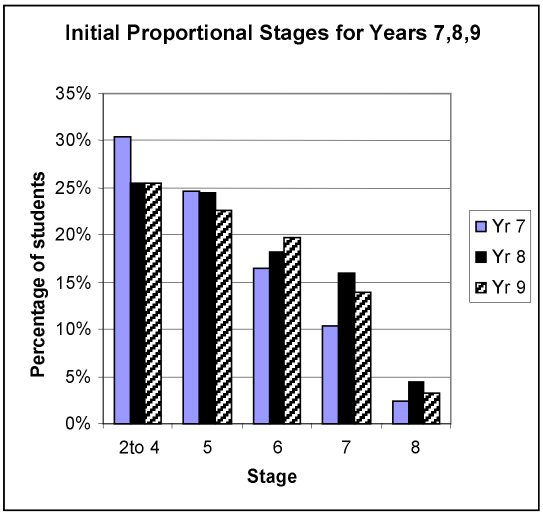


Figure 7.2. Initial and final stages on multiplicative strategies for years 7, 8, and 9.

On multiplicative strategies, year 8 had the highest proportion of students at the top stage, advanced multiplicative. The highest proportion of year 9 students was at Stage 6, early multiplicative.

Comparison of Proportional Stages

More students were not given this scale or were assessed as being at Stage 1 than were for additive strategies or multiplicative strategies. The percentages of students not included in the following figure were: year 7 – 15% initially and 7% finally; year 8 – 11% initially and 4% finally; and year 9 – 15% initially and 9% finally. The remaining students were assessed as being at Stage 2–4, 5, 6, 7, or 8, as shown in Figure 7.3.



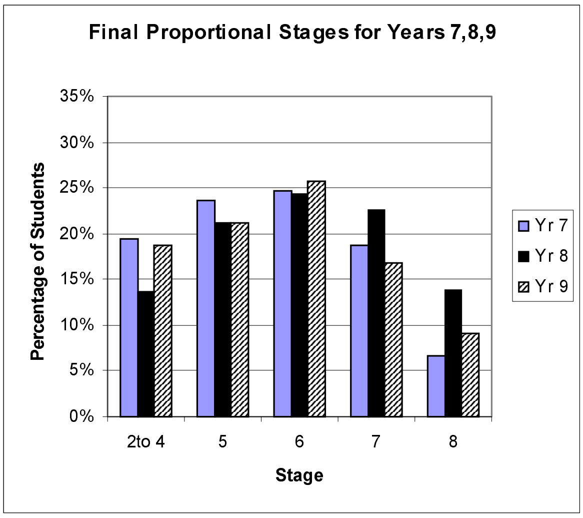


Figure 7.3. Initial and final stages on proportional strategies for years 7, 8, and 9

On this scale, the highest percentage for all year groups was initially at Stages 2–4, equal sharing. By the final analysis the majority of students in all years were at Stage 6, early multiplicative. Only Stages 7 and 8, early proportional and advanced

proportional, involve proportional thinking. A larger percentage of year 8 than year 9 students reached these stages. Only students assessed on Form C could score at Stages 7 and 8.

Summary

A larger percentage of year 8 students reached the top stages than did of year 7 or year 9 students on all three strategy scales. It is reasonable to expect year 8 to show greater progress than year 7. While it is unusual for year 9 students to do less well, this may be related to the preponderance of low decile students in the sample.

Chapter 8 – Overall Comparison of Gains Between 2001 and 2002

Very different numbers of schools participated in the project in 2001 and 2002. One issue behind the percentage of students gaining in 2001 was the low stage of students' initial competence on tasks that were new to both students and teachers. This seems to have been less true in 2002, possibly because of changes in the assessment questions or possibly because major holes in students' knowledge had been addressed before assessment. The following table gives the percentages of students at the ceiling and gaining at least one stage, for scales that were used in both 2001 and 2002. Percentages for 2001 are taken from Irwin and Niederer (2002). Percentages for 2002 for years 7 and 8 are from those presented in this report. There were some differences in items on the assessment in the two years.

Scales of knowledge for fractions, decimals, and percentages are not included. These were combined into one scale in 2001. The scales of knowledge for numeral identification and the order of numbers were a single scale in 2001 and three separate scales in 2002.

Table 8.1. Comparison of percentages of year 7 and 8 students gaining at least one stage in 2001 and 2002.

	Additive Strategies	Multiplicative Strategies	Proportional Strategies	Knowledge of Place-value
2001	46%	45%	44%	54%
2002	35%	49%	57%	57%

(N for 2001 = 1,871, Maximum N for 2002 = 11,849, but varying numbers were assessed on these scales on both occasions. See text for these variations.)

Fourteen secondary schools participated in 2002, while 12 had participated in 2001, with only 10 returning their data. The percentage of students gaining at least one stage in year 9 is reported here. In 2001 data were returned for only seven students in year 10, so no comparison can be made.

Table 8.2. Comparison of percentages of year 9 students gaining at least one stage in 2001 and 2002.

	Additive Strategies	Multiplicative Strategies	Proportional Strategies	Knowledge of Place-value
2001	45%	44%	43%	51%
2002	27%	38%	43%	49%

(N for 2001 = 1,451, Maximum N for 2002 = 1,446, but varying numbers were assessed on these scales on both occasions. See text for these variations.)

Thus gains among year 9 students were less marked in 2002 than they were in 2001 for both additive and multiplicative scales, but similar for proportional strategies and knowledge of place-value.

An important statistic to compare is the number of students reaching the top stage. The following table suggests that the lower percentage of gains in additive strategies could be related to the higher percentage of students already at ceiling. The lower percentage of 2002 students reaching ceiling on multiplicative strategies and place-value require different explanations.

Table 8.3. Percentages of year 9 students at the top stage in 2001 and 2002.

	Additive Strategies	Multiplicative Strategies	Proportional Strategies	Knowledge of Place-value
2001	21%	31%	8%	21%
2002	33%	24%	9%	13%

Several factors might be related to the lower percentage of students gaining in multiplicative strategies and knowledge of place-value scales in 2002 despite the project running for a longer period. A likely factor is the difference in the assessment forms, which did not assess students at the top two levels on these scales.

Chapter 9 – Views of Participating Teachers and Facilitators

The organisers of this project indicate that it is primarily a professional development project for teachers, aided by in-school facilitation. One purpose of the interviews in this evaluation was to see how teachers and facilitators viewed this and other aspects of the project.

To gain the views of teachers in the project, semi-structured interviews were held in person with intermediate school teachers in two parts of the country, similar interviews were held by telephone with leaders of the project in three secondary schools in other parts of the country, and an anonymous questionnaire was sent to all intermediate and secondary schools known to be taking part in the project. Initial questions for the interviews and questionnaires are given in Appendix F. In the interviews, initial questions were followed by other relevant questions that arose from the participants' responses.

To canvas the views of facilitators, telephone interviews were held with all intermediate and secondary school facilitators. The topics covered in these interviews are also given in Appendix F. Some of these facilitators also worked with year 7 and 8 teachers in full primary schools. However, facilitators whose main role was with primary schools that also had year 7 and 8 classes were not contacted.

Telephone interviews were very informative and were, in the author's view, as useful as face-to-face interviews. They often lasted for about an hour, as informants talked about their views. Questionnaires were sent out after these interviews, in an attempt to reach a wider audience. Due to unforeseen circumstances, these questionnaires went out later in the year than expected and no attempt was made to elicit further responses.

Teachers' Views

Teachers' views were sought on the following issues

- the assessment;
- main benefits and any disadvantages of the project for themselves and for their students;
- what aspects of the Numeracy Project they taught and for how long;
- use of grouping;
- contacts with their facilitators.

In the chapters below, information from the face-to-face and telephone interviews has been included with the information from questionnaires.

Questionnaires were sent to 439 teachers in all schools known to have teachers of years 7–10 participating (51 schools). Ninety-four teachers returned questionnaires. As questionnaires went out late in the year, due to unexpected circumstances, this was a reasonable return. While I cannot know if this is a representative sample there is no reason to believe that it is not. There were responses from schools in all 10 deciles for years 7 and 8 and from secondary schools in deciles 1, 3, 4, 8, and 9.

Table 9.1. Year level, school's year in the project, and background of students of teachers who returned questionnaires.

	Years 7–8	Years 7–8	Years 9–10	Years 9–10	Not given
Year in project	1st	2nd	1st	2nd	
Number of responses	50	16	10	18	
Number of teachers reporting students in the project previously	15	12	4	8	31

Assessment forms that teachers used

Form B was the assessment form used most commonly. More than two thirds of the teachers reported using more than one form for an individual child, with some reporting the use of a different form on the initial and final test. Several teachers made negative comments about the length of the testing and about the fact that different stages were represented on the different forms. Four secondary teachers reported primarily using Form A. These may have been in classes for students with particular needs, or it could indicate that the strategy windows (See Appendix A) were inappropriate for this age group.

Intermediate school teachers who were interviewed reported:

Most on Form B, couple of A and a couple of C forms.

Year 8 teacher numeracy leader.

B was the most prevalent … For some I had to switch between A and B. A was too easy and B was too difficult. This is particularly a problem at intermediate level.

Year 8 teacher.

Mostly C and a handful of B, but in retesting all will be C.

Year 8 teacher of high ability class.

This last comment gave a foretaste of difficulties in evaluating the success of the project, as there was not a smooth transition between the stages on the different forms.

Forty-two percent of the teachers reported that at least some of their students had been in the Numeracy Project in the previous year while only 14% were sure that none of their students had been involved previously. The rest of the teachers were either unsure if any students had been involved or did not respond to this item. Unfortunately, this information was not available for the assessment data.

In interviews, teachers presented more views on the assessment interviews. In 2001, secondary school teachers had reported that these interviews were the best aspect of the project, but in 2002 most comments were on the length of the assessment interviews. The two comments below show a different understanding of what testing was required.

The new form of the test that required every student to be tested on all items took longer than last year's test did and took more than the allotted time. This was particularly true of Form C, yet it was worth going through to the end with all students.

Year 9 teacher and numeracy leader, second year in the project.

The tests took far too long. We reduced the testing, using Form B on all but the top class for whom we used Form C.

Year 9 teacher and numeracy leader, second year in project.

For further views on the length of testing, see the chapter on negative views of the project.

Main benefits for students

Teachers of years 7 and 8 and of years 9 and 10 both saw the main benefit as being increased student confidence in using different strategies. Some also commented that they developed increased confidence in their students' abilities. Some expressed shock at the lack of knowledge of incoming students, wondering how they had slipped through the system so far. Secondary school teachers saw advantages in the specificity of the project for different students and the emphasis on the enjoyable hands-on activities and discussions. Year 7 and 8 teachers appreciated the increased emphasis on number and the chance to plug gaps in students' knowledge. One commented on the benefit of "children thinking about their thinking".

We have seen a definite improvement in the maths learning of the students in our school.

Year 7 and 8 teacher, second year in project.

Main benefits for teachers

For themselves, teachers of year 7 and 8 students appreciated the increased accuracy of their assessing and teaching and their increased personal knowledge, especially of strategies. Secondary school teachers appreciated their deeper knowledge of their students. Some also reported on their improved personal understanding of how to teach mathematics at this level.

The majority of teachers (56%) said that the project had changed their awareness of students' mathematical skills and strategies, but surprisingly 22 teachers (24%), 13 of whom were in their first year of using the project, said that the project had not changed their understanding of their students' mathematics. Similarly, most teachers said that the project had changed their view of how to teach mathematics but 18 teachers (19%) said that it had not changed their view of how to teach mathematics.

Nine of these teachers were in the project for the first time. It was not possible to tell whether these teachers were already teaching in a manner consistent with the project or whether they did not believe that they should change their traditional methods. Personal interviews showed that there were some teachers who did not believe that a change in their teaching method was warranted, although they were attempting to implement some parts of the project.

A question on whether or not the project had changed their own understanding of mathematics at this level brought a few responses indicating that they had learned new aspects of mathematics or needed to learn new aspects. However, many teachers, especially secondary school teachers, interpreted this question as referring to the students' mathematics rather than their own. One teacher, who did understand the intention of the question and was aware of her shortcomings, wrote:

Mostly I was taught using algorithms, and my ability to find ways of teaching using strategies was certainly challenged … I believe that this programme relies on the skill and confidence of the teacher. It is therefore my recommendation that workshops are offered to boost teacher confidence and skills. I don't think that learning alongside students is very valuable. Students may get more out of the programme when teacher's skills are secure.

Year 7 and 8 teacher, second year in the project.

In interviews, intermediate teachers talked about the value of the assessment.

It's been a lot of work and the initial assessment took a lot of time, but not more than taking Running Records. … Throughout my teaching, although we were shown how to do a reading assessment, we were never shown a maths assessment. The only time was when I took a New Entrant class and we did "Checkout".

Year 8 teacher and project leader. first year in project.

One very positive, more general remark made in interview was:

I've really enjoyed it. I've always struggled with teaching maths. I'm not a mathematical person at all. My strength is language. I've struggled with that constructivist thing of starting with materials and how much of that you should do, which materials to use. We've never had guidance with that. At College we didn't do a lot on how to teach maths. The project has given us some real guidance on where to start from and where to work to. That's what I've enjoyed the most. And the assessment shows you where the kids are in different areas of numeracy. The books give you ideas to follow that are going to move them to the next level. You can see that happening with the kids … So I've enjoyed it. It has been extra work but I don't think it has been extra work for extra work's sake. There has actually been a huge amount of progress for me and for my kids. When they comment at the end of the term that was one thing that they said they had gained in. A huge number said that had gained in decimals and percentages. It had finally clicked for them. They were life-long skills that they would be able to use.

Year 8 teacher, first year in project.

Main disadvantages for students

About half the teachers saw no disadvantages for students. Those who did mention disadvantages thought bright students were likely to get bored. Others thought that all students were likely to become confused about the relationship of mental strategies and algorithms. Some year 8 and year 9 teachers mentioned anxiety about testing or seeing the comparison with their second test as a disadvantage.

Sometimes the skill we were trying to assess was lost in the contextual setting particularly with fractions … [The] interview part was most important to see about plus/minus/ multiply/dividing strategies.

Year 9 teacher, second year in project.

Those that really enjoyed maths disliked doing this numeracy work, most preferring doing a problem that they then research and solve.

Year 8 teacher, second year in project.

Main disadvantages for teachers

Some teachers felt pressured to teach in a manner that they were not convinced was appropriate. One commented:

I saw value in the project [but] I did not like the very rigid programme and secretly deviated from time to time. [The] programme's good, needs streamlining, BY ME!

Year 9 teacher, second year in project.

The main negative aspect seen by teachers at both levels was the time taken and extra workload involved in the project. This included planning and organisation and the related stress. Nearly 80% of the teachers made comments on this, some of them with strong feeling. One secondary school project leader, who was new to the project but in a school in its second year, wrote:

As a manager of the project I found it very time-consuming arranging the sets of materials to be prepared, e.g., to be laminated and cut out, arranging for photocopying, staff relief, and time tables for the testing. The data entry is cumbersome … Another difficulty … was having changes in staff and having to do re-training with staff for re-testing, that happened with four out of 10 classes! The project was worthwhile but I would never commit to testing all year 9 and year 10 again – just focus on the low achievers.

Year 9 and 10 project leader, first year in project.

Several year 7 and 8 teachers wrote of difficulty in understanding the project. One appreciated that any programme that suited the needs of the students and teachers was likely to succeed, but wanted evidence of the benefits of this particular project.

I feel that with sufficient curriculum knowledge, teacher eagerness, resources (as supplied by this programme) and a balanced programme, suited to the needs of the children – any project will succeed, but teacher security and faith in the programme with background data on its success is very important.

Year 7 and 8 teacher, first year in project.

As well as asking to interview teachers with a variety of views, the researcher also talked to enthusiastic ones and ones who were not happy with the project. The following quotation came from a less enthusiastic teacher.

Initially I wasn't enjoying the Numeracy Project at all. I felt that way because I was learning all about the new school as well as everything else. I found the Numeracy Project initially quite difficult to implement, possibly because I'm a new teacher and possibly because one minute you're being told to teach things one way, and suddenly in a new school you aren't using any of the things you were learning [in the last school]. What made it easier for me was that in our team we had decided to cross-group so rather than having to plan for three very diverse levels I now have to plan for three groups that are within cooee of each other. … Firstly, I have to teach mathematics everyday because you are getting new children in your room … I can't opt out and go back to the old ways of doing things. … For two terms I felt ill-prepared and very uncomfortable … I think we needed to continue on with the old way of doing maths, perhaps measurement, geometry, and things that aren't so much part of the numeracy part of things, and once we had more of a grasp of the Numeracy, start it rather than throwing out everything that was old and trying to introduce everything new straight off.

… I found it quite stressful. We are all at varying levels of experience in our team, me being new, one of my team is over 60 and has been teaching for many many years … we all found it quite difficult. One day we were doing an activity and it was above kids' level of ability, the next day we did another and they found it too easy, so it was hard finding the activities and pitching it to the right level, and encouraging the kids to see that what they were doing as real maths. Many of my more able students looked at it and thought, "this is baby stuff". (When I asked this teacher if she could place herself at a stage on the project she said that she was now a part-whole thinker but had not been when the project began.)

Year 8 teacher, first year in project.

Grouping

Teachers were asked if they grouped students occasionally, most of the time, or all of the time. Responses were fairly evenly divided among these three categories, with 40% indicating that they grouped most of the time. A quarter (27%) of the teachers from both the primary and the secondary sector said that they grouped seldomly or occasionally, and 33% reported that they grouped their students all of the time. More teachers of years 7 and 8 grouped most or all of the time, while those who used groups occasionally were more likely to be secondary teachers. Three quarters (73%) of the teachers who responded to this item used three or four groups. Six teachers reported using between five and eight groups.

Most of the teachers who were interviewed said that they grouped their students in varying ways for different parts of the project. The secondary numeracy coordinators interviewed had also grouped, but said that not all teachers in their schools had been happy with this. A comment made on one of the questionnaires was:

Secondary teachers are not used to grouping like primary teachers are, so it is hard to convince them of the need to group teach and not whole class teach. Maybe some demos on group teaching would help. Need to get more documentation into secondary schools around the Numeracy Framework, bit too waffley at the moment.

Year 9 teacher, second year in project.

Attention to different scales in the project

Because, in 2001, it had been impossible to tell if difference in progress in different strategies was related to different emphasis on teaching, teachers were asked to estimate the amount of time that they had spent teaching additive, multiplicative, and proportional strategies. Teachers answered this question either with the number of weeks or in more general terms. Most were able to distinguish what they had spent time on and the different emphases for different groups. The percentages of teachers indicating that they spent five or more weeks or "lots" of time teaching each strategy strand was as follows: additive strategies – 53%, multiplicative strategies – 45%, and proportional strategies – 51% (multiple responses were possible). Some teachers indicated that they spent more time with their lower groups on additive strategies and more time with their upper groups on proportional strategies. This indicates that teachers were conscientious in covering all scales. Because questionnaires were anonymous it is not possible to match students' progress with the amount of time that teachers reported spending on each scale.

Teachers were also asked which knowledge scale or scales they emphasised. Many indicated spending time on all. Where a topic was emphasised more than others, that topic was fractions for the year 7 and 8 teachers and decimals and percentages for the secondary school teachers. The fact that a higher proportion of secondary teachers reported emphasising number sequences may relate to the fact that there were some teachers of quite low ability classes among those that responded, shown by the fact that they chose to test using Form A.

Table 9.2 Percentages of teachers reporting emphasising the knowledge strands (multiple responses were accepted).

Knowledge Strand	Year 7–8 teachers (n=66)	Secondary teachers (n=28)
Number sequence	36%	62%
Fractions	76%	52%
Decimals and percentages	61%	70%
Grouping and place-value	67%	63%

Teachers' views of facilitators

Our facilitator (named) was FAB! What a key component to the successful adoption of a new programme.

Year 7 and 8 teacher, first year in project.

What an interesting project! Maths was never a strength of mine but thanks to our facilitator and all the resources given to us to support the project, I feel a lot more confident about teaching maths. Kia ora!

Year 7 teacher, second year in project.

Teachers' views of facilitators ranged from negative to very positive, like those quoted above, with most teachers being very grateful for the help. In response to the question of what aspects were found to be useful, responses could be categorised as "all"; modelling, class visits, and demonstrations; guidance and clarity of procedure; and activities and resources which were helpful and well explained. Demonstrations and modelling in classes were the aspects appreciated by most teachers. The suggestion for improvement made by the greatest number of teachers was for more support and more modelling from the facilitator. Four teachers found the presentations overwhelming and three found the time allocation did not meet their needs.

Comments about facilitators were not elicited from teachers who were interviewed, as there would be no anonymity in these cases. However two people commented on the different styles of the facilitators in 2001 and 2002, in one case stating a strong preference and in the other case just indicating that it took a while to get used to the new facilitator. These comments indicated that teachers had formed a close, trusting relationship with the facilitators. Facilitators were more than impersonal deliverers of a project.

Summary of teachers' views

Although there were both positive and negative responses to questions about the project, there were many more positive responses than negative ones from both year 7 and 8 and secondary school teachers.

Concerns about the project related to time and workload, and for some, a desire to continue teaching algorithms.

Teachers expressing positive attitudes embraced new ideas and liked the focus on number and the practical resources. They generally enjoyed more one-to-one time with students, especially that provided by the interviews. They enjoyed seeing the confidence in students who were allowed to use different strategies for solving problems.

Facilitators' Views and Experiences

Being a facilitator was not a straightforward job, as indicated in the comments from teachers. It required a number of interpersonal skills, areas of knowledge, insights, and individual characteristics. The goal of a facilitator was to change the practice and beliefs of other adults in some way. This usually requires some knowledge of the existing beliefs and practices of those adults. In many ways, it is similar to being a teacher and also as difficult as being a teacher.

Facilitators differed from the teachers in that they had all chosen to take on this role rather than having a senior administrator in a school decide that they would be involved.

The 14 facilitators who worked with teachers on this project came from a variety of backgrounds. Some had had many years of experience advising other teachers and others were new to this role. Their teaching backgrounds ranged from having been teachers of children in junior classes (aged 5 and 6) to having been secondary school teachers. It can be assumed that they all differed in their interpersonal skills.

Despite this, it is reasonable to presume that all were successful in helping some teachers change, although no one claimed to have seen all the teachers that they worked with change as much as they would have liked. Telephone interviews were held with 14 facilitators for both the Intermediate Numeracy Project and the secondary Numeracy Project. Interviews lasted up to one hour. Questions were asked about:

- the skills that were required in order to be a good facilitator;
- differences in how they provided services to their schools;
- the level of commitment to the project that they saw different teachers as having;
- the effect that they thought that the project had had on teachers' understanding of their students, teachers' understanding of how mathematics is best taught, and teachers' understanding of mathematics;
- what they saw the essence of the Numeracy Project to be.

As these interviews became conversations, they intentionally strayed into other areas that either party wanted to talk about. The question about the essence of the Numeracy Project came up in one of the early discussions and was asked of all facilitators interviewed.

Facilitators' views are separated by topic. In some cases there was a difference in the views of those who worked with year 7 and 8 teachers and those who worked with secondary school teachers. These are separated where appropriate.

Facilitators' views of the qualities needed for their role

Although this question was asked toward the end of the interview, the views expressed reflected what the facilitators had chosen to do with the schools that they worked with. Facilitation appears to have varied because of the beliefs of these people. It is interesting how often the characteristics that facilitators thought they needed in their jobs were the same as those that they thought were essential for teachers in the project. The need for facilitators to listen to teachers was parallel to the need for teachers to listen to students. Identifying teachers' needs was similar to teachers identifying students' needs. One facilitator quoted her supervisor in the Advisory Service as having a motto of "valuing what they are doing and then suggesting bite-sized chunks that they can manage". This value is as useful for teaching young students as for teaching adults.

Facilitators gave the following main points as essential for their job:

1. Listening to teachers and starting from where they are. This was among the most common traits seen as essential for facilitators.

   (The following quotations were from transcriptions from notes made during and after the telephone call and may not be the facilitator's exact words.)

   We need to be able to listen.

   We need to be open, have to adapt to where teachers are.

   Don't tell. That would be disastrous.

2. Respecting the adults you are working with. This frequently mentioned quality could be considered to be a result of listening to teachers. It involved an essential part of working with adults known to most adult educators.

   These are knowledgeable adults, so don't assume that you can tell them how to change the world.

   Value the knowledge that teachers bring, and be sure that they know that you value it.

   Don't make assumptions about the teachers; don't talk down to them.

3. Reassuring teachers.

   They don't know they are doing a good job, in the isolation of one class, in one school.

4. Helping teachers develop new understanding. This is the next step beyond respecting teachers. This was expressed as:

   Resisting the temptation to tell. As in teaching, I answer a question with a question that makes them think. It is important for teachers to make the learning journey.

   Valuing what they are doing and then suggesting bite sized chunks that they can manage.

   Encouraging teachers through things.

5. Having experience and skill as a teacher.

   Facilitators making this comment included those who valued their experience as junior class teachers, with experience in responding to individual needs and grouping children for teaching purposes. Other facilitators who made this comment found that their ability in getting alongside adolescents was particularly useful, or their experience as the head of a secondary mathematics department, which meant that they could speak as equals to others in the same position. One facilitator who had been a secondary teacher but was facilitating in intermediate

schools said that her experience enabled her to reassure intermediate teachers that secondary schools would value the work that they were putting in on numeracy. One facilitator mentioned this quality as having a solid classroom pedagogy and skill in classroom management. For another facilitator this included the willingness to take risks in front of teachers. For other facilitators this was mentioned as a personal characteristic like:

I love teaching kids. I always ask for the bottom group.

You need to be creative, think on your feet.

6. Having enthusiasm. Interestingly, only facilitators who worked with intermediate level teachers mentioned this characteristic. We cannot imply that facilitators who worked with secondary schools did not value this attribute.

7. Being good at organisation of time and materials. This was mentioned by some as a quality that they brought to the job and by others as a quality that they had to develop on the job. For some it involved managing to meet schools' needs as well as possible while carrying out all of the other parts of their employment (none of these facilitators worked fulltime on the Numeracy Project). A complex version of the need for these skills was:

   Structuring situations to achieve the best result. E.g., ensuring budgets spent, managing relief teachers, ensuring adequate independent work for non-teaching groups.

8. Knowing mathematics. When not mentioned, this may have been another assumed characteristic. One reported this in terms of "having a strong curriculum knowledge" and another as "having a framework to understand kids' development of the understanding of mathematics."

9. Building trust with those in the school. Several facilitators mentioned the fact that they had worked with the school in previous years or that they knew the teachers through their local mathematics teachers association and this had been valuable in building up trust.

   It takes at least two years to understand a school, to build rapport, and to get them telling you that they have a problem.

10. Understanding the school systems and pressures on staff at different levels. This was mentioned most often by teachers who had had to learn about different systems since becoming a facilitator. One found it difficult to talk with principals as she had always been in a situation in which she was responsible to principals. Another was learning about the complexities of life in intermediate schools, which were different from the full primary schools she had taught in.

11. Understanding of the Number Framework and the research underlying it. Two facilitators mentioned this characteristic as an area in which they continued to learn on the job.

When facilitators were asked what they were still learning, many commented that they were improving in all of the skills that they had indicated that were necessary for

a facilitator. One said that it was the best professional development for herself that she had ever been involved in. Others gave thoughtful statements about broad aspects of the project:

I keep thinking about ways of implementing the project in secondary schools. What are the barriers and what makes it work in secondary schools? I want to identify what has to come first in schools, or helps if it does. For example, [a helpful precursor is] schools that are already committed to increasing attention to numeracy.

I am interested in the algebra stuff – what is critical in teaching it.

This list of characteristics required by a facilitator would make an excellent set of criteria for applicants for such positions to consider.

Service provided to schools by the facilitators

There were set services that facilitators were to provide for schools. However, how and when they provided these services was open to some negotiation. The standard pattern for work with schools in their first year of the project was to meet with principals and numeracy lead teachers, and then to hold workshops on the Number Framework, the assessment, addition and subtraction strategies, multiplication and division strategies, fractions and ratio strategies, the transfer to algebra, and long-term planning. There were visits to teachers in schools on four or five occasions.

The service that facilitators provided almost always showed a response to the needs of the schools they were working with. The nature of facilitators' jobs varied from being responsible for one or two urban schools to being responsible for eight or 12 year 7 and 8 classes in country areas where there could be students from year 4 to year 8 in one class. All facilitators reported that they had a different procedure for working with schools in the second year of the project from those in the first year of the project. The most common procedure for schools in the second year was to provide a session on the new format of the assessment and then leave the schools to organise how the project was carried out. Facilitators were available when consulted, often sent emails with new materials, and visited staff, demonstrating, watching teachers, and advising as was requested.

Some facilitators said that the form of service they gave to schools in their second year of the project varied, depending on the needs of the school. New teachers from these schools were either invited to join in workshops for new schools or helped through visits to their classes. One facilitator reported holding a separate session for all new teachers in February when teachers were introduced to the Number Framework and the assessment, then helping these new teachers in their classrooms. Some facilitators thought that new teachers had received less attention than they would have liked. Comments included:

Three beginning teachers didn't get as much help as they needed. They were included in seminars in the new schools and had a visit each, plus help from within the school, but needed more.

They missed out by not being part of the initial enthusiasm in the year the school first became involved.

Facilitators varied in how they interacted with schools. The majority of workshops were held after school. Many were reported to be of about two hours duration. Some facilitators held workshops in a teacher-only day before school began and again in the breaks between terms. One facilitator held a long after-school seminar, from 3:45–7 p.m., with pizza brought in. However, he found this unproductive, and vowed not to hold after-school sessions again, so moved to holding half-day workshops in school time, visiting teachers' classes in the other half-day. Some facilitators held seminars with clusters of schools while others worked with individual schools, a decision often influenced by geography. Most facilitators did all of the presenting. However, in one case, teachers were asked to take responsibility for the later workshops, so that each teacher prepared a lesson for a particular strategy and presented it to the others in the cluster. The facilitators involved in this approach reported that it set up a good model for future professional development in their area.

The length of time that schools spent on different topics, as reported by facilitators, differed. Some year 7 and 8 classes in full primary schools spent all of the second term on additive strategies and place-value, moving to multiplicative strategies only in the third term.

Facilitators' judgement of teachers' response to the project

A framework of teachers' responses to professional development, proposed by Claxton and Carr for the Learning in Science Project (Claxton and Carr 1991), was presented to facilitators. They were asked to think about how the teachers that they had worked with fitted within this framework.

Claxton and Carr suggest that when teachers are requested to make major changes in their underlying philosophy of teaching they may respond in a number of ways. One reaction might follow another, although there is no evidence that any teacher goes through all such reactions. These different reactions include *entrenchment* and *opposition* ("it's nothing to do with me", p. 7), beginning to think that the change is a *possibility*, *dabbling* with some of the aspects of the new project, *agreeing* that the idea may be a good one while wondering how practicable it is. Some teachers show *commitment*, or deciding to "go for it" (p. 7) while still holding private misgivings. Subsequent stages may be *clarification* in which teachers think through new processes, *introspection* in which they search for ideas or advice on the change, *planning*, *experimentation*, perhaps some *deflation* when things don't work as hoped or even anger at the "people 'whose stupid idea this was'" (p. 8). This might lead to disappointment, then reappraisal, *recuperation*, new commitment or *reaffirmation*, and *extension* of the ideas to new fields. The teacher may "fall into the trap of *evangelism* … and adopt the role of preacher in the staff room" (p. 8). Or the teacher may move to what Caxton and Carr call a more helpful stage of seeing the *limitations* of an approach and eventually having the new approach *permeate* all of their teaching.

Facilitators were given the main points of this framework and asked how the teachers that they worked with might fit. For many facilitators, the differences between the initial commitment and the later permeation were unclear. However, most found that they recognised the teachers that they worked with in this framework.

The following table is based on the facilitators' reports of the teachers, in the first or second year of the project, showing each of these reactions. Not all teachers were included in this classification.

Table 9.3. Teachers seen at each of Caxton and Carr's reaction stages, with regards to a curriculum reform in project schools.

	Year 7 and 8		Year 9 and 10	
	1st year	2nd year	1st year	2nd year
Number of teachers	147	61	11	61
Entrenchment or opposition	2%	5%	9%	8%
Possibility, dabbling, or agreement	22%	38%	45%	38%
Commitment, clarification introspection, and planning	45%	20%	36%	13%
Experimentation, deflation, recuperation, reaffirmation, and extension	12%	5%	0%	13%
Evangelism	2%	3%	0%	3%
Limitation (seeing where the project applied and where it did not apply	2%	11%	0%	15%
Permeation	14%	18%	9%	10%

For several schools, the majority of teachers were seen as being in the same category. For one intermediate school in the second year in the project, 14 out of 21 teachers (67%) were judged to be in the last two categories. In another intermediate school also in the second year of the project, 17 out of 21 teachers (81%) were seen as dabbling. Different facilitators served these schools. Schools also reported differences with the same facilitator. In one intermediate school in the first year of the project, 11 out of 22 teachers (50%) were judged to be dabbling, while in another with the same facilitator with 14/20 teachers (70%) were judged to be at the commitment stage and others at both ends of the continuum. In one secondary school in the second year, all teachers were seen as at the permeation stage.

There is no certainty that different facilitators viewed these categories in the same way. The figures given above are only suggestive. Facilitators did appear to be in agreement about the categories of "entrenchment or opposition" with a larger proportion of secondary school teachers than year 7 and 8 teachers being seen in these categories. A larger proportion of year 7 and 8 teachers than secondary school teachers were viewed as having incorporated the teaching philosophy of the project into their wider teaching philosophy. The majority of all teachers were seen to be at the stages of possibility, commitment, and experimentation. A higher percentage of teachers at both levels in the second year in the project were seen as being at the most advanced stages, which involved integrating the essence of the Numeracy Project into their general teaching philosophy.

This leads to the following question: What is the essence of the Numeracy Project that teachers might have integrated into their teaching philosophy? Teachers were not asked this, but facilitators were.

The essence of the Numeracy Project

This issue was not one of the initial interview questions, but came up in conversation in relation to some of the modifications that individual facilitators had made. It related to what they would be sure to include if other things had to be compromised.

Over half of the facilitators indicated that the essential aspect of the project was listening to students and, as a result of that, knowing how to help them move forward. Five facilitators mentioned the Number Framework as essential, for helping teachers know where to go next. However, two other facilitators thought that it was not essential. Other frameworks might be devised that were also useful. Some thought that strategies were useful, but one indicated that there were other things to move onto in secondary school mathematics. Two facilitators pointed to having students think critically about mathematics as a goal closely related to listening to students and knowing how to move them on.

Grouping of students and the assessment of students provided by the project were seen by some facilitators as essential and by others as not essential. While three facilitators mentioned grouping as essential three also said that it was not essential, so long as individual students' needs were appreciated and met. It was seen as an outcome of listening to students and attempting to meet their needs, not as the essence of the project. Similarly, four facilitators mentioned the diagnostic assessment as essential, as a way of getting teachers to listen to students, while three said that it was not essential for the main aim of listening to students. A sensitised teacher could listen to students as they worked and remember whose thinking had various qualities, even if they were secondary teachers teaching 150 different students a day.

Only one facilitator indicated that the essence or core of the project was improving teachers' confidence and capability. This was interesting in view of the Ministry's proposal that this was primarily a professional development project for teachers. It indicated that most saw the benefit of the professional development directly in teachers' capability to listen to and help students. It is likely that this specific skill was seen as the most important aspect of improving teachers' competence.

Final comments

Despite facilitators' comments about essential and non-essential aspects of the Numeracy Project, several volunteered comments, when asked if there was anything else that they wanted to say. The following comments caught the essence of these views.

I'm even more impressed with the project.

I still believe it is the best thing for developing numeracy strategies, thinking critically about mathematics.

Chapter 10 – Summary and Implications for Further Research

This report shows that the Numeracy Project is valuable for years 7–9, as was the project in 2001 (Irwin and Niederer, 2002). In addition, this evaluation demonstrates the need of a programme of this type for some year 10 students. Between 30% and 57% of students gained at least one stage on the different scales presented to them, in a part of the Number Framework where advancement is more difficult than it is for young children. There was some evidence in 2002 that all strategy scales were taught, whereas in 2001 this evidence was lacking.

The project was successful in enabling all but a small percentage of students to use part-whole strategies in addition. This is an essential basis for other work with numbers. Another aspect that older students appear to have learned quickly, if taught, is the reading of larger digits and knowledge of the numbers coming before and after them. Smaller proportions of students became multiplicative or proportional thinkers. Further teaching of strategies in these fields is needed.

Research is needed on appropriate methods for enabling older students to become multiplicative and proportional thinkers within a year. Several facilitators have experimented with different ways of advancing thinking in these areas more quickly than by taking them through the Numeracy Project steps, which were written for younger children.

There are other aspects of the project besides teaching of multiplicative strategies that appear to be more appropriate for younger children than for these older students. One such aspect is that the assessment forms appear to be more relevant to teachers of younger children than for older students. These use a quick test of additive strategies to see which form to use for each student. This again appears to be more appropriate for younger students than for students in this age range. The suggestion that a quick test of understanding of place value be used for all age groups seems appropriate.

A study of the ability of students to generalise principles learned in the Numeracy Project to a range of problems showed students in the project to score significantly higher than similar students not in the project. This was a test of algebraic, or pre-algebraic thinking, which should enable these students to apply these skills to secondary school algebra.

An unanswered question is whether students will use this flexible knowledge in secondary school mathematics, and in algebra particularly. We do not know if secondary school teachers will help students build a more abstract understanding of algebra upon the knowledge that they have acquired in this project. In order to answer this question, it would be necessary to work with teachers who are introducing algebra in secondary school, to see how they are introducing the subject and the extent to which it build on students' existing knowledge.

In the report on the 2001 cohort of students (Irwin and Niederer, 2002) attention was drawn to the difficulty that students in this age range had with fractions. The assessment of fractions was changed markedly for 2002, so it is not possible to comment on whether or not this cohort of students was more able in the area.

The use of different forms for assessment and the lack of reliability in teachers' assessments have drawn attention to problems in using the assessments, both for teachers' understanding of their students and as a method of evaluating the project. The test of pre-algebraic thinking has provided one independent measure of the effectiveness of the Numeracy Project. In future, other measures, such as asTTle tests, need to be used for evaluation of the project.

Teachers' responses to the project were generally favourable, although some found changing their manner of teaching difficult or unnecessary. Facilitators rated the teachers whom they worked with as ranging from resistant to having the main themes of the project integrated it into the way in which they thought about teaching. Part of the teachers' ambivalence may relate to the relationship of the Numeracy Project to other aspects of the mathematics curriculum (Ministry of Education, 1992) and especially to the more traditional secondary school components and the developing NCEA examinations. The relationship of the project to other strands of the curriculum could be the subject for further development and research. At some point, the Numeracy Project needs to be more closely integrated into a revised version of the mathematics curriculum.

Interviews with facilitators show them to be a highly skilled group, sensitive and responsive to the needs of the teachers whom they work with. Their understanding of the Numeracy Project appears to be a deep one and enables them to keep to the essence of the project, as they see it, while knowing where to be flexible with teachers when appropriate. They understand the difficulty of changing teachers' established practices and appear to have shown patience in the face of this challenge. A measure of the close relationship that they have built up with teachers is reflected in teachers' resistance to a change of facilitators.

Despite the successes of this project, it has not yet been shown to be successful for all students in all topics. We need to continue to experiment with ways of helping the least successful students, whether their lack of success is related to socioeconomic status, ethnicity, or specific difficulties in mathematics that require remedial help.

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Appendix A – Numeracy Assessment Forms A, B, and C

NumPA Form A Individual Assessment Sheet

* denotes cards needed # question booklet needed

Child's name: Date: Teacher:

Strategy window	Response	Strategy window	Response
Get eight counters for me.	Yes No	Four counters and three counters.	

Operational Strategy Questions

Addition and Subtraction #

(1) Here are 8 counters. Here are 4 more counters. How many counters are there altogether?

(2) I have 9 lollies, and I eat 3 of them. How many lollies do I have left?

(3) I have 13 marbles, and I lose 5 of them. How many marbles do I have now?

Stage 0 Emergent	**Stage 1** One-to-one Counting	**Stage 2** Counting from One on Materials	**Stage 3** Counting from One by Imaging	**Stage 4** Advanced Counting

Comments

Knowledge Questions

Forward Number Word Sequence (FNWS) *

(4) Start counting from 1. I'll tell you when to stop *(at 32)*.

What's the next number after …? The next number after 2 is 3. So if I say 2, you say 3. Now, what is the next number after …?

(5) 5 (6) 9

(7) 13 (8) 19 (9) 12 (10) 15

(11) 29 (12) 46 (13) 69 (14) 80

Stage 0 Emergent FNWS	**Stage 1** Initial FNWS up to 10	**Stage 2** FNWS up to 10	**Stage 3** FNWS up to 20	**Stage 4** FNWS up to 100

Comments

Backward Number Word Sequence (BNWS) *

(15) Count backwards from 10. I will tell you when to stop *(at 0).*

(16) Count backwards from 24. I will tell you when to stop *(at 11).*

What number comes before …? The number that comes before 2 is 1. So if I say 2, you say 1. Now, what number comes before …?

(17)	*3*	*(18)*	*5*	*(19)*	*9*	*(20)*	*8*
(21)	*16*	*(22)*	*20*	*(23)*	*17*	*(24)*	*11*
(25)	*47*	*(26)*	*13*	*(27)*	*70*	*(28)*	*31*

Stage 0 Emergent BNWS	Stage 1 Initial BNWS up to 10	Stage 2 BNWS up to 10	Stage 3 BNWS up to 20	Stage 4 BNWS up to 100

Comments

Numeral Identification *

What is this number?

(29)	*0*	*(30)*	*3*	*(31)*	*5*	*(32)*	*9*	*(33)*	*8*
(34)	*6*	*(35)*	*1*	*(36)*	*4*	*(37)*	*2*	*(38)*	*7*
(39)	*10*	*(40)*	*13*	*(41)*	*19*	*(42)*	*11*	*(43)*	*16*
(44)	*12*	*(45)*	*66*	*(46)*	*43*	*(47)*	*80*	*(48)*	*38*
(49)	*137*	*(50)*	*702*						

Stage 0 Emergent Numeral Identification	Stage 1 Numerals to 10	Stage 2 Numerals to 20	Stage 3 Numerals to 100	Stage 4 Numerals to 1000

Comments

Grouping and Place Value #

Tell me how many dots are in each picture

(51) Five dots and two dots? How many more to make ten dots?

(52) Five dots and four dots? How many more to make ten dots?

(53) Ten dots and four dots?

(54) Ten dots and seven dots?

(55) Six sets of ten dots?

Stage 0–1 Non-grouping with fives and within ten	Stage 2–3 With fives and within ten	Stage 4 With tens

Comments

NumPA Form B Individual Assessment Sheet

* denotes cards needed # question booklet needed

Child's name: Date: Teacher:

Strategy window	Response	Strategy window	Response
Four counters and three counters		Nine counters and seven counters	

Operational Strategy Questions	Stage 4 Advanced Counting	Stage 5 Early Additive Part-Whole		
Addition and Subtraction # (1) I have 8 counters under here, and I'm putting some more counters under here. Altogether, there are 13 counters now. How many are under here? (2) You have 37 lollies, and you eat 9 of them. How many have you got left? (3) In the bush, there are 48 kiwi and 25 weka. How many birds are there altogether?	Comments			
Multiplication and Division # (4) Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether? If I planted 15 more trees, how many rows of 5 would I have then? (5) What is 3 × 20? If 3 × 20 = 60, what is 3 × 18? (6) What is 8 × 5? If 8 × 5 = 40, what is 16 × 5?	**Stage 2–3** Count from One	**Stage 4** Advanced Counting	**Stage 5** Early Additive Part-Whole	**Stage 6** Advanced Additive Part-Whole
	Comments			
Proportions and Ratios # (7) Which of these cakes have been cut into thirds? Here are twelve jellybeans to spread out evenly on top of the cake. You eat one third of the cake. How many jellybeans do you get? (8) Suppose there were 28 jellybeans to spread evenly on a cake and you took three-quarters of the cake. How many jellybeans would you get?	**Stage 1** Unequal sharing	**Stage 2–4** Equal Sharing	**Stage 5** Early Additive Part-Whole	**Stage 6** Advanced Additive Part-Whole
	Comments			

Knowledge Questions	Stage 2 FNWS to 10	Stage 3 FNWS to 20	Stage 4 FNWS to 100	Stage 5 FNWS to 1000	Stage 6 FNWS to 1 000 000
Forward Number Word Sequence (FNWS) * (9) Start counting from 10. I will tell you when to stop (*at 32*). For each number I show you, tell me the number that comes just after it, that is, the number that is one more. Like, if I show you 4, you say 5. (10) 12 (11) 17 (12) 29 (13) 63 (14) 99 (15) 209 (16) 490 (17) 999 (18) 3049 (19) 63 079 (20) 989 999	Comments				

Backward Number Word Sequence (BNWS) *

(21) Start counting backwards from 23. I will tell you when to stop (*at 10*).

For each number I show you, tell me the number that comes just before it, that is, the number that is one less. Like, if I show you 4, you say 3.

(22) 13 (23) 19 (24) 30 (25) 76 (26) 100 (27) 401

(28) 680 (29) 900 (30) 2400 (31) 30 700 (32) 603 000

Stage 2 BNWS to 10	Stage 3 BNWS to 20	Stage 4 BNWS to 100	Stage 5 BNWS to 1000	Stage 6 BNWS to 1 000 000

Comments

Fractional Numbers *

(33) Here are some fractions. Say each fraction as I show it. Put each number with the picture that matches it. There will be two fractions left over.

(33) Put these fractions *(from question 33)* in order from smallest over here to largest over here. *(If correct, ask)* Why do you think one-quarter is less than one-third?

(34) Pizza Picasso cuts their pizzas into sixths. You buy eight-sixths *(pointing to* $\frac{8}{6}$ *)*. How much pizza would you get?

Stage 2–3 Non-fractions of regions	Stage 4 Assigned unit fractions	Stage 5 Ordered unit fractions	Stage 6 Co-ordinated numerators and denominators

Comments

Grouping (including place value) #

Tell me how many dots are in each picture.

(36) Five dots and two dots? How many more to make ten dots?

(35) Five dots and four dots? How many more to make ten dots?

(38) Ten dots and four dots?

(39) Ten dots and seven dots?

(40) Six sets of ten dots?

At the Ten Bank, they only have $10 notes and $1 coins. How many $10 notes would you need to make these amounts of money?

(41) $83 (42) $230 (43) $6,074 (44) $78,900

How many $100 notes would you need to make these amounts?

(45) $78,921 (46) $2,050,000

Stage 0–1 Non-grouping with fives and within ten	Stage 2–3 With fives and within ten	Stage 4 With tens	Stage 5 Tens in 100	Stage 6 Tens and hundreds in whole numbers

Comments

NumPA Form *C* Individual Assessment Sheet

* denotes cards needed   **#** question booklet needed

Child's name: ____________ Date: ____________ Teacher: ____________

Strategy window	Response	Strategy window	Response
Nine counters and seven counters		53 – 26 = ☐	

Operational Strategy Questions				
Operational Strategy Questions **Addition and Subtraction #** (1) Sandra has 394 stamps. She gets 79 stamps from her brother. How many stamps does she have then? (2) Hone has $403 in his bank account. He takes out $97 to buy a new skateboard. How much money is left in his account?	**Stage 6** Advanced Additive Part-Whole Comments			
Multiplication and Division # (3) Here is a forest of trees. There are 5 trees in each row, and there are 8 rows. How many trees are there in the forest altogether? If I planted 15 more trees, how many rows of 5 would I have then? (4) What is 3×20? If $3 \times 20 = 60$, what is 3×18? (5) What is 8×5? If $8 \times 5 = 40$, what is 16×5? (6) There are 24 muffins in each basket. How many muffins are there altogether? (7) At the car factory, they need 4 wheels to make each car. How many cars can they make with 72 wheels?	**Stage 4** Advanced Counting	**Stage 5** Early Additive Part-Whole	**Stage 6** Advanced Additive Part-Whole	**Stage 7** Advanced Multiplicative Part-Whole
	Comments			
Proportions and Ratios # (8) Suppose there were 28 jellybeans spread evenly on a cake and you took three-quarters of the cake. How many jellybeans would you get? (9) Suppose there were 35 jellybeans spread evenly on a cake and you took three-fifths of the cake. How many jellybeans would you get? (10) It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies? (11) There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class is boys?	**Stage 5** Early Additive Part-Whole	**Stage 6** Advanced Additive Part-Whole	**Stage 7** Advanced Multiplicative Part-Whole	**Stage 8** Advanced Proportional Part-Whole
	Comments			

Knowledge Questions

Forward and Backward Whole Number Word Sequence *

For each number I show you, tell me the number that comes just after it, the number that is one more. Also tell me the number that comes just before it, the number that is one less.

(12) 17 (13) 30 (14) 63 (15) 100 (16) 129 (17) 407
(18) 840 (19) 2400 (20) 3049 (21) 63 079 (22) 603 000 (23) 989 999

Stage 4 FNWS/BNWS to 100		Stage 5 FNWS/BNWS to 1000		Stage 6 FNWS/BNWS to 1 000 000	

Comments

Fractional Numbers *

(24) Here are some fractions. Say each fraction as I show it. Put each number with the picture that matches it. There will be two fractions left over.

(25) Put these fractions *(from question 33)* in order, from smallest over here to largest over here. *(If correct, ask)* Why do you think one-quarter is less than one-third?

(26) Pizza Picasso cuts their pizzas into sixths. You buy eight-sixths *(pointing to $\frac{8}{6}$).* How much pizza would you get?

(27) Here are some fractions ($\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{6}{9}$, $\frac{7}{16}$.) Put them in order, from smallest over here to largest over here.

Stage 2–3 Non-fractions of Regions		Stage 4 Assigned unit fractions		Stage 5 Ordered unit fractions		Stage 6 Co-ordinated Numerators and denominators		Stage 7 Equivalent Fractions		Stage 8 Ordered fractions	

Comments

Decimals and Percentages *

Say each decimal as I show it to you.

(28) 0.8 (29) 0.39 (30) 0.478

(31) Put these decimals in order, from smallest over here to largest over here.

(32) Round 7.649 to the nearest tenth.

(33) Round 2.38501 to the nearest hundredth.

(34) What is 1.25 as a percentage?

(35) Name 37.5% as a decimal.

Stage 4 Emergent decimal identification		Stage 5 Decimal identification		Stage 6 Ordered decimals		Stage 7 Rounded decimals		Stage 8 Decimal conversions	

Comments

Grouping and Place Value

At a bank, they only have $10 notes and $1 coins. How many $10 notes would they need to make these amounts of money?

(36) $63 (37) $238 (38) $6,074 (39) $78,916

How many $100 notes would you need to make these amounts?

(40) $78, 921 (41) $2,000,000

(42) How many tenths are in all of this number? *(4.67)*

(43) How many hundredths are in all of this number? *(2.592)*

Stage 4 With tens		Stage 5 Tens in 100		Stage 6 Tens and hundreds in whole numbers		Stage 7 Tens, hundreds, and thousands in whole numbers		Stage 8 Tenths, hundredths, and thousandths in decimals	

Comments

Appendix B – Percentages of Year 7 and 8 Students at Each Stage on Initial and Final Assessment.

Strategy Scales

Additive strategies

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
NA	1%	1%	1%	1%
1	0%	0%	0%	0%
2	1%	1%	1%	0%
3	1%	1%	1%	0%
4	30%	14%	25%	10%
5	42%	44%	39%	37%
6	24%	40%	34%	51%

Multiplicative strategies

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given	4%	4%	3%	2%
1	11%	3%	4%	1%
2–4	30%	19%	22%	11%
5	25%	24%	28%	20%
6	17%	25%	26%	32%
7	10%	19%	17%	34%

Proportional strategies

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given	4%	3%	3%	2%
2–3	6%	2%	8%	2%
4	27%	15%	26%	14%
5	30%	25%	25%	21%
6	24%	32%	18%	24%
7	10%	23%	16%	23%
8	2%	7%	4%	14%

Easier Knowledge Scales

Forward Number Word Sequence

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given or 0	1%	2%	1%	1%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	1%	1%	1%	0%
4	11%	6%	8%	4%
5	48%	34%	43%	26%
6	37%	58%	47%	68%

Backward Number Word Sequence

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
0	2%	3%	2%	3%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	1%	1%	1%	0%
4	15%	7%	11%	5%
5	45%	32%	41%	26%
6	36%	56%	44%	65%

Whole Number Identification

Stage	Initial Year 7	Final Year 7	Year 8 Initial	Year 8 Final
Not given	95%	96%	96%	97%
0	0%	0%	0%	0%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	1%	1%	1%	0%
4	3%	3%	3%	2%

More Complex Knowledge Scales

Knowledge of Fractions

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given	4%	3%	4%	2%
2–3	19%	5%	12%	3%
4	29%	15%	26%	12%
5	25%	31%	26%	26%
6	15%	26%	19%	28%
7	5%	10%	7%	13%
8	3%	9%	5%	16%

Knowledge of Decimals and Percentages – (only on Form C)

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given	70%	58%	59%	46%
2–3	0%	1%	0%	1%
4	5%	4%	4%	3%
5	11%	13%	15%	13%
6	6%	9%	10%	11%
7	4%	8%	7%	12%
8	2%	8%	5%	15%

Knowledge of Grouping and Place-value

Stage	Year 7 Initial	Year 7 Final	Year 8 Initial	Year 8 Final
Not given	1%	2%	1%	1%
0–1	3%	1%	2%	1%
2–3	15%	5%	12%	3%
4	34%	22%	30%	16%
5	28%	31%	28%	27%
6	10%	17%	13%	19%
7	5%	10%	8%	14%
8	4%	11%	6%	19%

Appendix C – Test of Pre-algebraic Manipulation in Arithmetic.

Task A

Jason works out problems like 47 + 25 and 67 + 19 in his head.

Problem	Jason's working	Your Explanation of Jason's Working
47 + 25	50 + 22 = 72	
67 + 19	66 + 20 = 86	

1) Explain Jason's working in the spaces above.
2) Show how Jason makes these calculations.

	Write your working in the spaces below
97 + 56	
268 + 96	
4613 + 987	

Task B

Kate works out problems like 183 – 94 and 87 – 48 in her head.

Problem	Kate's working	Your Explanation of Kate's Working
87 – 48	89 – 50 = 39	
183 – 97	186 – 100 = 86	

Explain Kate's working in the spaces above.

2) Show how Kate makes these calculations.

	Write your working in the spaces below
74 – 28	
262 – 96	
3421 – 289	

Task C

Kiri and Josh show how they work out problems like 3 x 88 in their heads.

	Kiri's way	Josh's way
3 x 88	3 x 90 – 2	3 x 90 – 6

1) One method is wrong. Decide which it is and explain why the method is wrong.

2) Show how to use one of the methods to correctly work out these problems.

Write your working in the spaces below

7 x 99	
9 x 989	
25 x 9996	

Task D

Work out the number that goes in each box.

Write your working in the spaces below

$\square$	+	26	=	431	
1758	=	$\square$	+	651	
$\square$	–	34	=	21	
237	–	$\square$	=	25	

Task E

Work out the number that goes in the first box in each problem.

2) Then write the answer to the calculation in the answer box.

Answer box ↓ | Write any working in this space

$5 \times 18 = 10 \times \square$ $\square$

$36 \times 25 = 9 \times \square$ $\square$

$48 \times 2.5 = \square \times 10 =$ $\square$

$27 \times 3\frac{1}{3} = \square \times 10 =$ $\square$

Task F

Work out the number that goes in each box.

Write any working in the spaces below

$$\frac{3}{4} = \frac{15}{\square}$$

$$\frac{21}{56} = \frac{\square}{8}$$

$$\frac{18}{30} = \frac{12}{\square}$$

$$\frac{\square}{4} = \frac{16}{10}$$

Appendix D – Acceptable Answers, Algebraic Rationale, and Analysis of Variance for Test of Pre-algebraic Manipulation in Arithmetic.

Acceptable Answers

A: Must add and subtract the same answer from each addend, and the number used must make the sum easier to do mentally. Accurate answers were not required.

1. 100 + 53
2. 264 + 100 or 270 + 94
3. 4,600 + 1,000 or 4,610 + 990

B. Must add the same number to each number (the minuend and subtrahend).

1. 76 – 30
2. 266 – 100
3. 3,434 – 300 or 3,432 – 290

C. Must subtract a multiple of the number not changed.

1. 7 x 100 – 7
2. 9 x 1,000 – 99 or 10 x 989 – 989 or 9 x 990 –9
3. 25 x 10,000 – 100

D. Must add given numbers in 3 and subtract given numbers in 1, 2 and 4. Calculation errors accepted if correct process used.

1. 405
2. 1,107
3. 55
4. 212

E. Must show evidence of multiplying one number and dividing the other number by the same number.

1. (5 x 2) x (18 / 2)
2. (36 / 4) x (25 x 4)
3. (48 / 4) x (2.5 x 4)
4. (27 / 3) x (3 1/3 x 3)

F. Correct answers found by multiplying.

1. 20
2. 3
3. 20
4. 24

Algebraic Rationale

Task A – *Additive Compensation*
$x + y = (x + a) + (y - a)$

Task B – *Equal Additions*
$x - y = (x + a) - (y + a)$

Task C – *Distributive Law*
$xy = xm - xn$ where $y = m - n$

Task D – *Equivalent Sums and Differences*

1. If $x + a = b$, then $x = b - a$
2. If $b = x + a$, then $x = b - a$
3. If $x - a = b$, then $x = b + a$
4. If $a - x = b$, then $x = a - b$

Task E – *Multiplicative Compensation*
$xy = (ax)(\frac{y}{a})$

Task F – *Proportional Reasoning*

1. If $\frac{a}{b} = \frac{an}{x}$, then x = b x n
2. If $\frac{an}{bn} = \frac{x}{b}$, then x = a
3. If $\frac{an}{bn} = \frac{c}{x}$, then x = c ÷ a x b
4. If $\frac{an}{bn} = \frac{x}{c}$, then x = c ÷ b x a

Analysis of Variance for Test of Pre-algebraic Manipulation of Arithmetic

Descriptive statistics

	Project status	DECILE	Mean	Std. Deviation	N
TOTAL	Out	Lower	7.44	4.81	244
		Higher	8.32	4.91	224
		Total	7.86	4.87	467
	In	Lower	8.00	5.56	159
		Higher	12.39	4.82	210
		Total	10.49	5.59	370
	Total	Lower	7.66	5.12	403
		Higher	10.29	5.27	434
		Total	9.02	5.36	837

Tests of Between-subjects Effects

Dependent Variable: TOTAL

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3264.193	3	1088.064	43.671	.000
Intercept	66679.315	1	66679.315	2676.249	.000
Num Proj	1093.287	1	1093.287	43.880	.000
Decile	1418.105	1	1418.105	56.917	.000
Num proj * decile	625.452	1	625.452	25.103	.000
Error	20754.376	833	24.915		
Total	92158.000	837			
Corrected Total	24018.569	836			

Estimated Margin Means

Dependent Variable: TOTAL

Project status	Mean	Std. Error	Decile	Mean	Std. Error
Out	7.88	.231	Lower	7.72	.254
In	10.19	.262	Higher	10.35	.240

Appendix E – Percentages of Year 9 and 10 Students at Each Stage

Strategy Scales

Additive Strategies

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	1%	1%	0%	0%
1	0%	0%	1%	1%
2	1%	0%	0%	0%
3	3%	1%	2%	1%
4	27%	16%	38%	21%
5	42%	48%	29%	42%
6	25%	33%	30%	35%
Total	100%	100%	100%	100%

Multiplicative Strategies

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	4%	4%	2%	1%
1	7%	3%	5%	2%
2–4	24%	17%	30%	18%
5	26%	21%	33%	31%
6	25%	33%	15%	25%
7	15%	23%	15%	23%
Total	100%	100%	100%	100%

Proportional Strategies

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	4%	5%	2%	1%
1	11%	4%	14%	7%
2–4	25%	19%	25%	21%
5	23%	21%	24%	28%
6	20%	26%	17%	18%
7	14%	17%	17%	15%
8	3%	9%	1%	10%
Total	100%	100%	100%	100%

Easier Knowledge Scales

Forward Number Word Sequence

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given or 0	1%	3%	0%	0%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	1%	1%	2%	0%
4	12%	6%	10%	3%
5	46%	33%	48%	27%
6	40%	58%	41%	70%
Total	100%	100%	100%	100%

Backward Number Word Sequence

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given or 0	1%	1%	0%	0%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	1%	1%	0%	0%
4	15%	8%	13%	4%
5	43%	31%	45%	24%
6	39%	59%	42%	72%
Total	100%	100%	100%	100%

Whole Number Identification

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	90%	91%	57%	59%
0	1%	0%	0%	0%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	0%	0%	3%	2%
4	9%	8%	40%	39%
Total	100%	100%	100%	100%

More Complex Knowledge Scales

Knowledge of Fractions

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	4%	5%	2%	0%
2–3	12%	4%	7%	4%
4	24%	14%	23%	10%
5	32%	31%	39%	32%
6	18%	27%	18%	29%
7	5%	6%	6%	8%
8	4%	13%	5%	17%
Total	100%	100%	100%	100%

Knowledge of Decimals and Percentages

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	70%	60%	65%	33%
2-3	0%	0%	1%	0%
4	4%	2%	4%	6%
5	10%	11%	8%	16%
6	6%	10%	7%	12%
7	5%	5%	6%	13%
8	4%	12%	9%	20%
Total	100%	100%	100%	100%

Knowledge of Grouping and Place-value

Stage	Year 9 Initial	Year 9 Final	Year 10 Initial	Year 10 Final
Not given	1%	3%	0%	0%
0–1	3%	1%	3%	0%
2–3	11%	3%	11%	5%
4	30%	18%	31%	17%
5	33%	36%	28%	30%
6	10%	17%	14%	19%
7	6%	8%	8%	16%
8	5%	13%	6%	13%
Total	100%	100%	100%	100%

Appendix F – Semi-structured Interviews and Questionnaires for Teachers, Numeracy Leaders, and Facilitators

1. Semi-structured interview used with teachers, both face-to-face and by telephone.

Tell me about how the Numeracy Project is going for you.

When did you do the initial testing? How did it go?

Tell me about your students' past experience in mathematics.

What aspects of the Numeracy Project did you teach, and when? If you can, give me an outline of how many weeks were spent on each aspect, and with each group.

Do you teach in groups? If so, how many groups, how often, and for what topics?

Did it matter if you were a new teacher or an experienced one?

What has been your contact with the facilitator?

2. Form Mailed Out for Feedback from Teachers in the Numeracy Projects in Years 7, 8, 9, and 10

Class level ________ Decile of School ___________

Your year in the Numeracy Project (circle) 1st 2nd

School's year in the Numeracy Project (circle) 1st 2nd

I. What assessment forms did you use? A B C
(Circle all forms used, then underline the form used most.)

Did you use more than one form for a single child?

II. What proportion of your students had had the Numeracy Project in the previous year?

III. What do you see as the main benefits of the project?

For your students?

For yourself?

IV. What negative aspects do you see the project as having?

For your students?

For yourself?

Did you group your students (circle):

Occasionally Most of the time All the time

How many groups? ______

V. Has the project changed your view of the mathematical skills of students at your level?

If so, how?

VI. Has the project changed your view of how to teach mathematics at this level?

If so, how?

VII. Has the project changed your understanding of mathematics needed at this level?

If so, how?

VIII. Indicate about how much time you spent teaching each of the strategies, either in weeks or words like “lots” or “none”. If appropriate, give for different groups, e.g., “bottom and middle groups – most of the time, top group – one week”:

Adding/subtracting strategies

Multiplying/dividing strategies

Ratio/fraction strategies

IX. Underline the knowledge strands you concentrated on

Number sequence, Fractions, Decimals and percentages, Grouping and place value

X. What aspects of the facilitator’s work did you find useful/not helpful?

What would you suggest for changes to the facilitator’s role?

XI. Other comments:

3. Framework for Interview with Facilitators, Who Were Emailed This in Advance of the Telephone Call

1. What form did your services take for each of the schools involved – how did you vary things to suit them, your time, other demands, etc.?

2. Where you think the teachers in each school fit on this list of stages in professional development (taken from Caxton and Carr, 1991) (e.g. At school A: two in opposition, one dabbling, seven committed or experimenting, and one evangelist)?

Schools			Categories
			Entrenchment or opposition (it's nothing to do with me, I know better)
			Possibility (maybe I could try), dabbling (lukewarm attempts), agreement (I like the idea, but have some reservations)
			Commitment, which may also include clarification (read about the project), introspection, (how does it fit my educational philosophy?), planning (I'll try this)
			Experimentation, which may involve some failed attempts and deflation, recuperation, reaffirmation, and extension
			Evangelism (the only true way)
			Limitation (it actually works better for some things and not others)
			Permeation (puts the ideas into practice whenever appropriate)

3. What effect do you think you had on:
   A – teachers' understanding of their students?
   B – teachers' understanding of how this mathematics is best taught?
   C – teachers' understanding of this mathematics?

4. What have been the skills that you have brought to this job?

5. What skills did you need and not have, or have to develop on the job?

6. What do you see as the essence of the Numeracy Project?

7. Anything else?