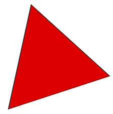
Platonics Discovered!

Construction 3D Models in Mathematics

It is very rewarding to discover and understand!

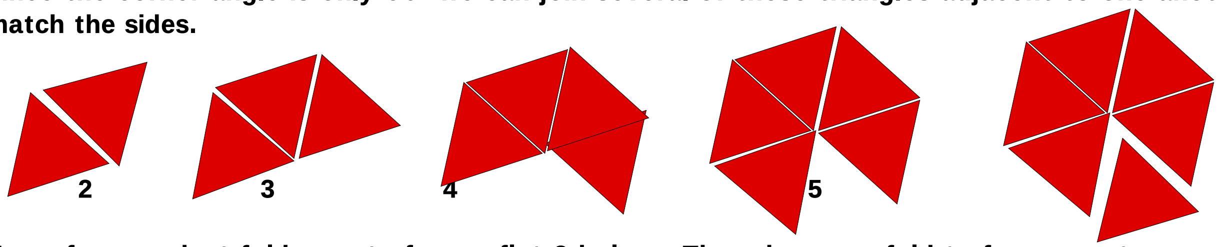
I have been making the Platonic solids with young children for a few years and it has finally dawned on me that in fact the tetrahedron, the octahedron, the icosahedron, the cube and the dodecahedron, in that order, are very unique in this world. No wonder Plato crowed about them!



I will try and explain.

The equilateral triangle is the first 2d shape we can make with straight lines. One and two being quite impossible.

Since the corner angle is only 60° we can join several of these triangles adjacent to one another taking care to match the sides.

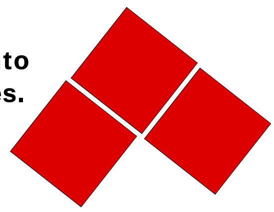


Two of course just fold over to form a flat 2d plane. Three however fold to form a vertex made from three triangles. All that is needed is a 4th triangle to complete the regular shape. This is the tetrahedron.

When we use four triangles, (4 x 60° which is still less than 360)they fold to form a vertex made from 4 triangles. Completing the shape always keeping to the "4 meeting at a vertex" rule creates an octagon. (cf The tetrahedron rule of "three meeting at a vertex")

Repeating this pattern with the 5 triangles leads us to a huge shape made from 20 sides, the icosahedron. The pattern is every vertex is made from 5 triangles.

When we get to 6, we find that 6x60 = 360 and the shape no longer folds into three dimensions but is a flat hexagon which only tessellates like floor tiles.

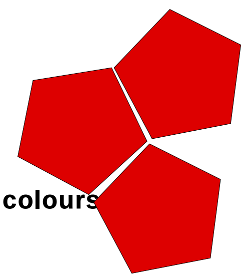


We have run out of options with the triangle! Now for the square.

Three fold to form a vertex. Completing the pattern creates a cube, or more correctly a hexahedron since it is made from 6 sides. There are no more options with the square because 4 x 90 = 360 which as before is a flat shape and will only tessellate.

The pentagon runs out just as quickly. Three fold to form a vertex and completing the pattern creates a dodecahedron. A curious 12 sided shape ideally suited to a 3d desktop calendar. Four pentagons exceed the 360° limit making a convex shape.

Three hexagons are also at the limit, three heptagons and beyond likewise. There are no other members to this set. It is simply unique! We have revealed a complete truth about the world we live in.



Extension 1
Make stellations for the sides, regular or different, brightly paint in two colours
Extension 2
It is worth counting the Vertices, the Faces and the Edges for each as well and uncover Euler's Rule.
Extension 3
Loping of corners reveals a miriad of other shapes which all have names.
Extension 4
The soccer ball and its inverse are made by joining pentagons around a hexagon or hexagons around a pentagon. These are dramatic when brightly painted with stellations

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